Parallel Rule Discovery from Large Datasets by Sampling

Wenfei Fan^{1,2,3}, Ziyan Han^{3}, Yaoshu Wang^{1}, Min Xie^{1}
Shenzhen Institute of Computing Sciences^{1} University of Edinburgh^{2} Beihang University^{3}
China^{1,3} United Kingdom^{2}
wenfei@inf.ed.ac.uk,hanzy@act.buaa.edu.cn,yaoshuw@sics.ac.cn,xiemin@sics.ac.cn

ABSTRACT

Rule discovery from large datasets is often prohibitively costly. The problem becomes more staggering when the rules are collectively defined across multiple tables. To scale with large datasets, this paper proposes a multi-round sampling strategy for rule discovery. We consider entity enhancing rules (REEs) for collective entity resolution and conflict resolution, which may carry constant patterns and machine learning predicates. We sample large datasets with accuracy bounds α and β such that at least α% of rules discovered from samples are guaranteed to hold on the entire dataset (*i.e.,* precision), and at least β% of rules on the entire dataset can be mined from the samples (*i.e.,* recall). We also quantify the connection between support and confidence of the rules on samples and their counterparts on the entire dataset. To scale with the number of tuple variables in collective rules, we adopt deep Q-learning to select semantically relevant predicates. To improve the recall, we develop a tableau method to recover constant patterns from the dataset. We parallelize the algorithm such that it guarantees to reduce runtime when more processors are used. Using real-life and synthetic data, we empirically verify that the method speeds up REE discovery by 12.2 times with sample ratio 10% and recall 82%.

CCS CONCEPTS

• **Information systems** → **Information integration**.

KEYWORDS

Rule discovery, data quality, sampling

ACM Reference Format:
Wenfei Fan^{1,2,3}, Ziyan Han^{3}, Yaoshu Wang^{1}, Min Xie^{1}. 2022. Parallel Rule Discovery from Large Datasets by Sampling . In *Proceedings of the 2022 International Conference on Management of Data (SIGMOD '22), June 12–17, 2022, Philadelphia, PA, USA.* ACM, New York, NY, USA, 15 pages. https://doi.org/10.1145/3514221.3526165

1 INTRODUCTION

Rule discovery has been a longstanding challenge for decades. To make practical use of rules such as functional dependencies (FDs [14]), conditional functional dependencies (CFDs [18]), denial constraints (DCs [4]) and matching dependencies (MDs [17]), we need to discover reliable rules from real-life datasets such that the rules can be frequently applied (*i.e.,* support) and frequently hold on the data (*i.e.,* confidence). While a number of discovery algorithms have been developed, the need for more scalable methods is evident. A recent study shows that it takes "days or longer" when mining FDs on a dataset with 100 attributes and 300K tuples [51].

The scalability problem of rule discovery becomes more staggering for rules beyond FDs. Recall that FDs $X \to Y$ are defined with *two relation atoms on a single table*. It has long been recognized that to accurately resolve conflicts (conflict resolution) and identify tuples that refer to the same tuple (entity resolution), one needs *collective* rules defined with multiple relation atoms to correlate information across tables [5, 22]. The cost of collective rule discovery is inherently exponential in the size of datasets since it requires to join multiple tables, no matter whether we use levelwise method [30] or depth-first search [69]. Worse yet, for rules that support constant patterns such as CFDs and DCs, the discovery cost inevitably increases since such patterns have to be enumerated.

To scale with large real-life datasets, a variety of sampling methods have been proposed [6–10, 13, 29, 31, 35, 42, 44, 46, 47, 52, 67]. The idea is to pick a representative sample $\mathcal{D}_s$ of a real-life dataset $\mathcal{D}$, and discover rules from smaller $\mathcal{D}_s$ instead of directly from large $\mathcal{D}$. However, these methods typically target rules that are defined on a single table with at most two relation atoms.

The sampling approach gives rise to several questions. Is there a sampling method that guarantees the accuracy of the rules Σ_s discovered from sample $\mathcal{D}_s$ *w.r.t.* the rules Σ mined from the entire dataset $\mathcal{D}$? That is, can we ensure that most rules in Σ_s hold on the dataset $\mathcal{D}$ and hence are also in Σ (precision), and moreover, most rules in Σ can be discovered from $\mathcal{D}_s$ and hence be covered by Σ_s (recall)? In addition, suppose that we discover Σ from $\mathcal{D}$ with thresholds σ and δ for support and confidence, respectively; then what thresholds and sampling size should we adopt for mining Σ_s from $\mathcal{D}_s$ to ensure a reasonable accuracy? How can we discover collective rules across multiple tables? If the rules support constant patterns like CFDs, how can we retrieve all such patterns when $\mathcal{D}_s$ inevitably drops constants from $\mathcal{D}$? When sampling alone does not suffice to scale with large datasets, *e.g.,* if the required samples are still large to meet an accuracy bound, is it possible to have a parallel algorithm that scales with the number of processors used?

Contributions & organization. This paper tackles these issues. As a testbed of our proposed method, we consider the class of Entity Enhancing Rules (REEs) [21, 22], which subsume CFDs, DCs and MDs as special cases, and are collectively defined across different tables with multiple relation atoms. As opposed to the previous rules, REEs specify rules for both entity resolution (ER) and conflict resolution (CR), and unify machine learning (ML) and rule-based methods by embedding ML models for ER, CR and similarity checking as predicates in logic rules. Like CFDs and DCs, REEs support

SIGMOD '22, June 12–17, 2022, Philadelphia, PA, USA
© 2022 Association for Computing Machinery.
ACM ISBN 978-1-4503-9249-5/22/06…$15.00
https://doi.org/10.1145/3514221.3526165

constant patterns of semantically related data. REEs are the rules underlying Rock, an industrial system for data cleaning. In this paper, we propose a novel sampling framework with optimization strategies for REE discovery. This said, the techniques can also be used to discover CFDs, DCs and MDs, which are special cases of REEs.

(1) Generic rules (Section 2). We introduce a representation of REEs [21, 22] by means of a *tableau* that specifies constant patterns, which helps us efficiently retrieve such patterns. We also identify practical ML models that can be plugged into REEs as predicates.

(2) REE *discovery problem* (Section 3). We formulate the discovery problem for REEs with sampling. We also characterize the accuracy of the set Σ_s of REEs discovered from sample $\mathcal{D}_s$, relative to the set of Σ mined from the entire dataset $\mathcal{D}$, in both *precision* and *recall*.

(3) A sampling strategy (Section 4). We propose a multi-round sampling strategy based on random-walk or BFS (Breath First Search) search to discover REEs independently in each sample. We prove an accuracy bound and deduce a sampling size for the sampling method with BFS. Under a given accuracy bound, we establish a connection between the thresholds of support and confidence for mining REEs on samples and the thresholds on the entire datasets, as a guidance for scaling samples and mining *valid* rules.

(4) Discovering REEs (Section 5). We present a parallel algorithm to discover REEs with sampling, denoted as PRMiner. We propose to train a reinforcement learning model and select semantically correlated predicates with the model to mine collective rules across multiple tables. We also show how we populate tableaux to efficiently retrieve constant patterns without enumerating numerous constant values. We prove that PRMiner is parallely scalable, *i.e.,* it guarantees to take less runtime when more processors are used.

(6) Experimental study (Section 6). Using real-life and synthetic data, we empirically verify the following. On average, (a) with sample ratio 10%, PRMiner speeds up rule discovery by 12.2 times with precision 90% and recall 82%, up to 93% and 85%, respectively; it improves random sampling by 7% in recall. (b) It scales well with the number n of processors, *e.g.,* it is 3.09 times faster when n varies from 4 to 20. (c) Employing ML correlation model in discovery improves the efficiency by 1.52 times on average, up to 3.77 times. (d) Constant pattern recovery further improves the recall by 2% on average, up to 4%.

Related work. We categorize the related work as follows.

Sampling methods. Various sampling methods have been explored for discovering rules. (1) *Random (uniform) sampling*, which is commonly used for its simplicity. In association rule mining, researchers mainly focused on deriving size bounds for a single sample to obtain high-quality approximation, *e.g.,* [7, 47, 67] adopt random sampling to derive the sample size with Chernoff bounds and union bound. The bounds are improved by using, *e.g.,* the central limit theorem [32, 41, 73] and hybrid Chernoff bounds [75]. For data quality rule, [44] uses uniform sampling to discover DCs using Chebyshev inequality, to estimate the number of rule violations and uses this estimate to derive appropriate parameters for rule mining. Unfortunately, [44] limits the discussion on bi-variable DCs on a single relation, and do not consider constant patterns. (2) *Focused sampling.* [35] and [6] utilize auxiliary structures, *e.g.,* agree sets or evidence sets, for rule discovery. Due to the high complexity, they adopt focused sampling to sample tuple pairs for constructing the desired structures. (3) *Progressive sampling.* To improve the loose bound of single random sampling, [8, 10, 13, 29, 31, 46, 52] adopt multi-iteration sampling, such that an initial random sample is first drawn and is then revised iteratively. Unfortunately, they fail to bound the sample size. In contrast, [57] mines frequent itemsets with size guarantee using the theorem of Rademacher Averages. (4) *Stratified sampling.* The method is adopted for mining association rules [42] on a single relation with size bound for estimating the support of itemsets, and answering aggregated queries [70] to minimize the sample size while satisfying an error bound. (5) *Heuristic sampling* for mining association rules by *e.g.,* [9]. Although it offers no guarantee, the strategies often achieve high accuracy.

This work differs from the prior. (a) To the best of our knowledge, this is the first sampling strategy with accuracy bounds for data quality rules that carry (i) more than two relation atoms across multiple tables, (ii) patterns of semantically related constants, and (iii) ML predicates. (b) We provide a bound on the sample size while guaranteeing precision and recall instead of error rates. (c) We establish the connection between support and confidence thresholds on the sample and the entire dataset. (d) We make a first effort to employ multi-round sampling in data quality rule discovery, to boost up the accuracy compared with single-round sampling.

Rule discovery. Efforts have been made on discovering data quality rules, classified as follows. (1) *Levelwise search.* Lattice traversal are widely adopted to discover FDs, *e.g.,* TANE [30], FUN [50], FD_mine [71]. Techniques are also developed to speed up the search, *e.g.,* agree sets are utilized in Depmine [45], sampling techniques are combined in HyFD [51] to prune non-FDs, DynFD [61] focuses on FD discovery in dynamic datasets, and SMFD [25] proposes a top-down framework to discover and validate FDs in a secured multi-party scenario. Levelwise methods are also used for CFDs and MDs, *e.g.,* CTANE [19] and tableau generation [26] for CFDs, and similarity thresholds [64] for MDs. (2) *Depth-first search.* Depth-first traversal is also applied for FD discovery, *e.g.,* DFD [2] mines FDs via random walk, FastFDs [69] improves Depmine by using different sets. Depth-first search is extended to CFDs and DCs, *e.g.,* FastCFDs [19] extends FastFDs to mine CFDs, while FastDC [12], Hydra [6], DCFinder [53] and ADCMiner [44] use evidence sets to mine *bi-variable* DCs, although by definition, DCs support multiple relation atoms [12]. (3) *Hybrid approaches.* HyMD [60] combines levelwise and depth-first approaches to mine MDs with pre-computed structures. MDedup [34] selects useful MDs discovered by HyMD. (4) *Learning-based approaches.* State-of-the-art learning methods have been used for rule discovery, *e.g.,* inductive learning in [23] and structure learning (*i.e.,* graphical lasso) in AutoFD [74], and the rule learning strategies in [33, 63]. More ER solvers can be found in [11].

This work differs from the prior work as follows. (1) Our multi-round sampling strategy is flexible and can be combined with state-of-the-art discovery algorithms, *e.g.,* DCFinder [53] and HyMD [60]. (2) We propose a strategy to discover collective rules across multiple tables using reinforcement learning. (3) We propose a method for efficiently retrieving constant patterns, to supplement sampling, which inevitably drops constants from the original datasets.

Parallel rule discovery. Several parallel algorithms are already in place for rule discovery. Massive parallelism is employed in [24, 39] to discover FDs. However, they do not take communication cost into consideration. [40] mines FDs in a distributed setting but it returns local FDs only. Communication cost is minimized in [58], which is extended from FastFD. A distributed framework is proposed in [59], which is capable of discovering both FDs and DCs, based on a set of primitives; however, the primitives do not support rules with constant patterns, ML predicates and multiple tuple variables; while in theory, the primitives can be extended to deal with these, it incurs an exponential cost in the number of tuple variables. SMFD [25] only focuses on how to enforce privacy constraints, although it discovers FDs based on the lattice structure in a distributed manner.

Different from the existing works, we develop a discovery algorithm that guarantees the parallel scalability, when both computational and communication costs are considered, and supports mining rules with constants, ML models and multiple tuples.

2 COLLECTIVE RULES WITH ML MODELS

We next present entity enhancing rules (REEs) defined in [21, 22].

Consider database schema $\mathcal{R} = (R_1, \ldots, R_m)$, where R_j is a schema $R(A_1 : \tau_1, \ldots, A_n : \tau_n)$, and each A_i is an attribute of type τ_i. We assume *w.l.o.g.* that each tuple t in D has an id attribute, which uniquely defines the entity that t represents. An instance $\mathcal{D}$ of $\mathcal{R}$ is a collection $(D_1, \ldots, D_m)$, where each D_i is a relation of R_i.

Predicates. Predicates are the atmoic formats of tuple relational calculus [3] and are used to represent correlations among attributes. They constitute the basic components of REE rules.

Definition 2.1: *We define* predicates *(i.e., atomic formulas) over a database schema $\mathcal{R}$ as follows:*

$$p ::= R(t) \mid t.A \oplus c \mid t.A \oplus s.B \mid \mathcal{M}(t[\bar{A}], s[\bar{B}]),$$

where $\oplus$ is either $=$ or $\neq$. Following tuple relational calculus (see, e.g., *[3]), (1) $R(t)$ is a* relation atom *over schema $\mathcal{R}$, where $R \in \mathcal{R}$, and t is a tuple variable* bounded by *$R(t)$. (2) When t is bounded by $R(t)$ and A is an attribute of R, $t.A$ denotes the A-attribute of t. (3) In $t.A \otimes c$, c is a constant in the domain of attribute A in R. (4) In $t.A \otimes s.B$, $t.A$ and $s.B$ are* compatible, i.e., *t (resp. s) is a tuple of some relation R (resp. R'), and $A \in R$ and $B \in R'$ have the same type. Moreover, (5) $\mathcal{M}$ is an ML classifier, $t[\bar{A}]$ and $s[\bar{B}]$ are vectors of pairwise compatible attributes of t and s, respectively.*

REEs. Employing predicates, we next define entity enhancing rules, which subsume most of existing data quality rules [21].

Definition 2.2: *An* entity enhancing rule *(*REE*) φ over a database schema $\mathcal{R}$ is a first-order logic formula and is defined as*

$$\varphi : X \rightarrow p_0,$$

where (1) X is a conjunction of predicates *over $\mathcal{R}$, and (2) p_0 is a predicate over $\mathcal{R}$ such that all tuples variables in φ are bounded in X. We refer to X as the* precondition *of φ and p_0 as the* consequence *of φ.*

Example 1: *Consider an organization database with two self-explained relation schemas:* Org *(*oid, org_name, zipcode, org_address, city, country*) in Table 1, and* Pers *(*pid, oid, persona_name, title, major, person_address, nationality*) in Table 2. Some example* REEs *over the database schema are given as follows.*

(1) φ_1 : $\text{Org}(t_a) \wedge \text{Org}(t_b) \wedge \mathcal{M}_{\text{Bert}}(t_a[\bar{A}], t_b.[\bar{A}]) \rightarrow t_a.\text{zipcode} = t_b.\text{zipcode}$. *Here we use the state-of-the-art ML model $\mathcal{M}_{\text{Bert}}$ [15] to check the semantic* similarity *of text attributes, where $\bar{A}$ denotes* (org_name, org_address) *in relation* Org. *Intuitively,* REE *φ_1 states that if two organizations have semantically similar names and addresses (checked by $\mathcal{M}_{\text{Bert}}$), then they have the same zipcode.*

(2) φ_2 : $\text{Pers}(t_a) \wedge \text{Pers}(t_b) \wedge \text{Org}(s_a) \wedge \text{Org}(s_b) \wedge t_a.\text{title} = t_b.\text{title} \wedge \mathcal{M}_{\text{Bert}}(t_a.\text{person\_name}, t_b.\text{person\_name}) \wedge \mathcal{M}_{\text{LP}}(t_a, s_a) \wedge \mathcal{M}_{\text{LP}}(t_b, s_b) \wedge s_a.\text{oid} = s_b.\text{oid} \rightarrow t_a.\text{pid} = t_b.\text{pid}$. *Here we use a* link prediction *model $\mathcal{M}_{\text{LP}}(t, s)$ [62, 66] to predict whether person t works in organization s (note that* Pers *might not record the fact "t works in s"). Intuitively, this* REE *collectively utilizes the information from both* Org *and* Pers *to identify two persons if they have similar names, same titles and works in same organizations. The* REE *is defined across two tables in terms of four tuple variables.*

(3) φ_3 : $\text{Org}(t_a) \wedge \text{Org}(t_b) \wedge X \rightarrow \mathcal{M}_{\text{addr}}(t_a.\text{org\_address}, t_b.\text{org\_address})$*, where $\mathcal{M}_{\text{addr}}$ is an ML model for checking the* closeness *of addresses, and $X = \bigwedge_{A_s \in \mathcal{T}} t_a.A_s = t_b.A_s$ and $\mathcal{T}$ denotes a designated set of attributes in* Org *(which are not shown in the simplified schema), including built-up area, located city, and neighborhood information. Here the conditions in X interpret the prediction of $\mathcal{M}_{\text{addr}}(t_a.\text{org\_address}, t_b.\text{org\_address})$ in logic. That is, $\mathcal{M}_{\text{addr}}$ predicts true because of the logic characteristics in X.*

Semantics. Consider an instance $\mathcal{D}$ of $\mathcal{R}$. A *valuation h* of tuple variables of φ in $\mathcal{D}$, or simply a valuation of φ, is a mapping that instantiates t in each $R(t)$ with a tuple in a relation D of R.

Valuation h *satisfies* a predicate p, written as $h \models p$, if the following are satisfied: (1) If p is a relation atom $R(t)$, $t \oplus c$ or $t.A \oplus s.B$, then $h \models p$ is interpreted as in tuple relational calculus following the standard semantics of first order logic [3]. (2) If p is $\mathcal{M}(t[\bar{A}], s[\bar{B}])$, then $h \models p$ if $\mathcal{M}$ predicts true on $(h(t)[\bar{A}], h(s)[\bar{B}])$.

For a set X of predicates, we write $h \models X$ if $h \models p$ for *all* predicates p in X. For an REE φ, we write $h \models \varphi$ such that if $h \models X$, then $h \models p_0$. An instance $\mathcal{D}$ of $\mathcal{R}$ *satisfies* φ, denoted by $\mathcal{D} \models \varphi$, if $h \models \varphi$ *for all* valuations h of φ in $\mathcal{D}$. We say that $\mathcal{D}$ *satisfies* a set Σ of REEs, denoted by $\mathcal{D} \models \Sigma$, if for each REE $\varphi \in \Sigma$, $\mathcal{D} \models \varphi$.

Example 2: *Continuing with Example 1, assume that $\mathcal{D}$ consists of two relations D_1 and D_2 of schemas* Org *and* Pers, *shown in Tables 1 and 2, respectively. Consider valuation h_2: $t_7 \mapsto t_a$, $t_8 \mapsto t_b$, $t_3 \mapsto s_a$ and $t_4 \mapsto s_b$. It satisfies* REE *φ_2, since p_3 and p_4 have similar names, the same titles and their working organizations are predicted to be the same by the link prediction model $\mathcal{M}_{\text{LP}}$.*

As another example, consider the valuation h_1 of REE *φ_1 that has mappings: $t_1 \mapsto t_a$ and $t_2 \mapsto t_b$. It helps us fix conflicting values in* zipcode *of t_1 and t_2, which have similar names and addresses.*

ML predicates. REEs can embed ML classifiers of the following.

- NLP models. REEs can embed language models, such as NER [27, 72], for text classification and semantic matching.
- ER models. One can use entity resolution and link prediction classifiers for multi-attribute record matching, *e.g.,* ditto [43], BertER [37] which return Boolean values.
- CR models. We can also plug in classifiers for data fusion and error detection *e.g.,* HoloClean [56] and HoloDetect [28].

tid	oid	org_name	zipcode	org_address	city	country
t_1	o_1	Guangzhou No One School	510375	Liwan Guangzhou Guangdong	GZ	CN
t_2	o_2	Guangzhou No.1 Middle School	510000	Liwan District, GZ, Guangdong	GZ	CN
t_3	o_3	Indiana U., Department of Biology	47401	Indiana Ave, IN, USA		USA
t_4	o_4	Indiana Univ., Computer Science	47401	IN	Bloomington	US

Table 1: Example Organization (Org) **relation** D_1

tid	pid	oid	person_name	title	major	person_address	nationality
t_5	p_1	o_1	Qiang Zhang	Teacher	math	Liwan, Guangzhou, GD	CN
t_6	p_2	o_1	Qiang Zhang	Teacher	math	Guangzhou	CN
t_7	p_3	o_3	Matthew Hahn	Prof.	Bioinformatics	Bloomington, IN, USA	USA
t_8	p_4		M. Hahn	Prof.	Bio.		

Table 2: Example Person (Pers) **relation** D_2

Pattern format. We call an REE expressed in $X \rightarrow p_0$ as its *regular format*. Equivalently, φ can be expressed in one-to-one corresponding *pattern format* $\varphi = (P \rightarrow Q, t_p)$ where P and Q are tuple attributes and operators used in X and p_0, respectively, and t_p is a pattern tuple, indicating how constant values and operators are applied in P and Q. To illustrate, consider a regular format REE:

$$\varphi : R(t) \wedge R'(s) \wedge t.A \neq s.B \wedge \mathcal{M}(t.C, s.D) \wedge t.E = c \wedge s.F \neq d \rightarrow t.G = e.$$

Its pattern format is expressed as: $\varphi = (P \rightarrow Q, t_p)$ where

$$P = [t.A, t.C, t.E, s.B, s.D, s.F, s.G, \oplus_1, \oplus_2],\ Q = [t.G],$$
$$t_p = (\oplus_1.\text{left}, \oplus_2.\text{left}, c, \oplus_1.\text{right}, \oplus_2.\text{right}, \bar{d}, \neq, \mathcal{M} || e),$$

and $\oplus_i$.left (resp. $\oplus_i$.right) denotes the left (resp. right) operand of $\oplus_i$. Note that the constant value of $t.E$ is directly assigned in t_p; similarly for $s.F$ and $t.G$ (a bar above indicates an inequality).

Note that we can easily *recover* a pattern format REE $\varphi = (P \rightarrow Q, t_p)$ to its regular format, *e.g.*, to recover X from P, (1) for each operator $\oplus_i$ in P whose assigned value in t_p is "=" (resp. "$\neq$"), we construct a predicate $p : \oplus_i.\text{left} = \oplus_i.\text{right}$ (resp. $p : \oplus_i.\text{left} \neq \oplus_i.\text{right}$) in X where $\oplus_i$.left and $\oplus_i$.right are the operands specified in t_p; (2) for each operator $\oplus_i$ in P whose assigned value in t_p is the ML model "$\mathcal{M}$", we construct a predicate $p : \mathcal{M}(\oplus_i.\text{left}, \oplus_i.\text{right})$ in X; and (3) for each attribute $t.A$ in P whose constant value c (resp. $\bar{d}$) is directly assigned in t_p, we construct a predicate $p : t.A = c$ (resp. $p : t.A \neq d$) in X. The recovery of p_0 from Q is similar.

Pattern tableau. When multiple REEs only differ in the constant values in constant predicates, they can be expressed concisely in one tableau REE in the form $\varphi_{\mathcal{T}} = (P \rightarrow Q, \mathcal{T}_p)$, where $\mathcal{T}_p$ is a *pattern tableau* consisting of a finite number of pattern tuples. For example, we can define a pattern tableau $\varphi_{\mathcal{T}} = (P \rightarrow Q, \mathcal{T}_p)$ using the same P and Q defined above, and $\mathcal{T}_p = \{t_p^1, \ldots, t_p^k\}$ where

$$t_p^i = (\oplus_1.\text{left}, \oplus_2.\text{left}, c_i, \oplus_1.\text{right}, \oplus_2.\text{right}, \bar{d_i}, \neq, \mathcal{M} || e_i).$$

The pattern tableau $\mathcal{T}_p$ can be classified into two categories: (1) *constant pattern*, if $\mathcal{T}_p$ contains at least one constant value, and (2) *variable pattern*, if $\mathcal{T}_p$ does not have any constant value. Later, we will use pattern tableau to retrieve constant patterns from the data.

3 RULE DISCOVERY WITH SAMPLING

We first formulate notions and sampling accuracy (Section 3.1), followed by the discovery problem with sampling (Section 3.2).

3.1 Preliminary

In the literature of rule discovery, the validity of rules is usually measured by two measurements, namely, *support* and *confidence*.

Support. Support measures how frequently a rule can be applied. To illustrate support, we first define an order on rules.

Definition 3.1: *Given two* REEs $\varphi : X \rightarrow p_0$ *and* $\varphi' : X' \rightarrow p_0$, *we say that* φ *has a* lower order *than* φ', *denoted by* $\varphi \preceq \varphi'$, *if* $X \subset X'$. *Intuitively,* φ *is less restrictive than* φ'.

We use the following notions. Given a predicate p, we define an REE φ_p to verify whether two tuples satisfy p: $R(t) \wedge R'(s) \rightarrow p$, where t and s (of relation schema R and R', respectively) are the tuple variables used in p. Let H_p be the set of valuations of φ_p in $\mathcal{D}$. We define the *support set* of p on $\mathcal{D}$, denoted by spset$(p, \mathcal{D})$, as

$$\text{spset}(p, \mathcal{D}) = \{\langle h(t), h(s)\rangle \mid h \in H_p \wedge h \models \varphi_p\},$$

i.e., the set of tuple pairs satisfying p. Similarly, given a conjunction X of predicates, we define the support set of X as follows:

$$\text{spset}(X, \mathcal{D}) = \{\langle h(t), h(s)\rangle \mid \forall p \in X(\langle h(t), h(s)\rangle \in \text{spset}(p, \mathcal{D}))\},$$

i.e., the set of all tuple pairs satisfying *all* predicates in X.

Given $\varphi : X \rightarrow p_0$, assume that H is the set of all valuations of φ in $\mathcal{D}$, and t_0 and s_0 are the tuple variables used in p_0. Then the *support set* of φ, denoted by spset$(\varphi, \mathcal{D})$, is defined as

$$\text{spset}(\varphi, \mathcal{D}) = \{\langle h(t_0), h(s_0)\rangle \mid h \in H \wedge h \models X \wedge h \models \varphi\}.$$

Definition 3.2: *The* support *of* φ *to quantify its frequency is*

$$\text{supp}(\varphi, \mathcal{D}) = |\text{spset}(\varphi, \mathcal{D})|.$$

Similarly we define the notions of supp$(p, \mathcal{D})$ and supp$(X, \mathcal{D})$.

For an integer σ, an REE is *σ-frequent* on $\mathcal{D}$ if supp$(\varphi, \mathcal{D}) \geq \sigma$.

One can verify that the above notation of support satisfies the anti-monotonicity, *i.e.,* if $\varphi \preceq \varphi'$, then supp$(\varphi, \mathcal{D}) \geq$ supp$(\varphi', \mathcal{D})$.

Example 3: *Let* X *be* Org$(t) \wedge t$.zipcode = 510375 *and* p_0 *be* t.city = *GZ. Consider two* REEs $\varphi : X \rightarrow p_0$ *and* $\varphi' : X' \rightarrow p_0$, *where* $X' = X \wedge$ Pers$(s) \wedge t$.oid = s.oid. *Clearly,* $\varphi \preceq \varphi'$ *since* $X \subset X'$. *Then* supp$(\varphi, \mathcal{D}) \geq$ supp$(\varphi', \mathcal{D})$, *since* spset$(\varphi, \mathcal{D})$ = spset$(\varphi', \mathcal{D})$ = $\{t_1 \mapsto t\}$, i.e., *anti-monotonicity. Although* t_1 *can join with two tuples,* t_5 *and* t_6, *in* φ', *it does not lead to a larger support.*

Confidence. Confidence indicates how often an REE $\varphi : X \rightarrow p_0$ has been found to be true, given that X is satisfied.

Definition 3.3: *Given an* REE $\varphi : X \rightarrow p_0$, *the* confidence *of* φ *on* $\mathcal{D}$, *denoted by* conf$(\varphi, \mathcal{D})$, *is defined to be* conf$(\varphi, \mathcal{D}) = \frac{\text{supp}(X \wedge p_0, \mathcal{D})}{\text{supp}(X, \mathcal{D})}$.

For a threshold δ, an REE is *δ-confident* on $\mathcal{D}$ if conf$(\varphi, \mathcal{D}) \geq \delta$.

Minimality. An REE $\varphi : X \rightarrow p_0$ over $\mathcal{R}$ is said to be *trivial* if $p_0 \in X$. In the rest of this paper, we only consider non-trivial REEs.

An REE $\varphi : X \rightarrow p_0$ is *left-reduced* on $\mathcal{D}$ if φ is σ-frequent, δ-confident and moreover, there exists no REE φ' such that $\varphi' \preceq \varphi$ and φ' is σ-frequent and δ-confident. Intuitively, it means that no predicate in X can be removed, *i.e.,* the minimality of predicates.

A *minimal* REE φ on $\mathcal{D}$ is a non-trivial and left-reduced REE.

Cover of rules. Consider a set Σ of minimal rules on $\mathcal{D}$.

We say that Σ *entails* another rule φ over $\mathcal{R}$ denoted by $\Sigma \models \varphi$, if for any instance $\mathcal{D}$ of $\mathcal{R}$, if $\mathcal{D} \models \Sigma$ then $\mathcal{D} \models \varphi$.

We say that Σ is *equivalent* to another set Σ' of rules, denoted by $\Sigma \equiv \Sigma'$, if $\Sigma \models \varphi'$ for all $\varphi' \in \Sigma'$ and vice versa.

We say that Σ is *minimal* if for all rules $\varphi \in \Sigma$, $\Sigma \not\equiv \Sigma \setminus \{\varphi\}$, *i.e.,* Σ includes no redundant rules.

Definition 3.4: *A* cover Σ_c *of* Σ *on* $\mathcal{D}$ *is a subset of* Σ *such that (a)*

symbols	notations
$\mathcal{D}$, $\mathcal{D}_s$	dataset, and samples of the dataset
φ	REE $X \rightarrow p_0$
Σ, Σ_s	the sets of REEs discovered from $\mathcal{D}$ and $\mathcal{D}_s$, respectively
α, β	bounds for precision(Σ, Σ_s) and recall(Σ, Σ_s), respectively
σ, δ	thresholds for support and confidence, respectively
RHS	the set of candidate consequences
$\mathcal{P}_0$	the set of relevant predicates

Table 3: Notations

$\Sigma_c \equiv \Sigma$, *(b) all rules φ in Σ_c are* minimal, i.e., *non-trivial and left-reduced. (c) Σ_c is* minimal, i.e., Σ_c *contains no redundant rules [3].*

Accuracy. Let $\mathcal{D}_s$ be a sample picked from $\mathcal{D}$, and Σ_s and Σ be the set of minimal rules discovered by the same discovery algorithm on $\mathcal{D}_s$ and $\mathcal{D}$, respectively. We quantify the effectiveness of sampling by *precision* and *recall*. Here precision, denoted by precision(Σ, Σ_s), reports the percentage of rules in Σ_s that also hold on $\mathcal{D}$, and recall, denoted by recall(Σ, Σ_s), is defined to be the percentage of rules in Σ that can be discovered from $\mathcal{D}_s$ and thus, covered by Σ_s.

3.2 Problem Statement

Assume that Σ_s and Σ are the set of minimal rules mined by the same discovery algorithm on $\mathcal{D}_s$ and $\mathcal{D}$, respectively. Clearly, Σ_s and Σ may contain an excessive number of rules that are not very relevant to users' applications and interests. To reduce such rules, we adopt the following strategies. (1) We pick an application-dependent set of candidate consequences p_0, denoted by RHS, which pertains to a particular application of users. (2) For each p_0 in RHS, we focus on discovering REEs φ: $X \rightarrow p_0$ such that $X \subseteq \mathcal{P}_0$ where $\mathcal{P}_0$ is a *subset* of semantically relevant predicates related to p_0. The *discovery problem with sampling* is as follows.

- *Input*: A database schema $\mathcal{R}$, a database $\mathcal{D}$ of $\mathcal{R}$, a consequence set RHS, the support threshold σ, the confidence threshold δ, the precision threshold α and the recall threshold β.
- *Output*: A cover Σ_s of REEs on a sample $\mathcal{D}_s$ picked from $\mathcal{D}$ such that (1) precision$(\Sigma, \Sigma_s) \geq \alpha$, recall$(\Sigma, \Sigma_s) \geq \beta$, and (2) for each REE $\varphi : X \rightarrow p_0$ in Σ_s, (a) $p_0 \in$ RHS; (b) $X \subseteq \mathcal{P}_0$, where $\mathcal{P}_0$ is a set of semantically relevant predicates related to p_0; and (c) φ is minimal and moreover, it is σ-frequent and δ-confident.

As verified by our industry collaborators, it is not very difficult for practitioners to select a few predicates in RHS since they often have a good understanding of their datasets and know their pain points. They only care about key attributes and are able to pick RHS by referencing the schema of datasets. For novice users, they can simply start with all predicates as RHS and then narrow down to what they need. A real-life case is deduplication in a particular dataset; in that case users simply choose the predicate t_0.id = t_1.id as the RHS predicate and only discover relevant ER rules.

The notations of the paper are summarized in Table 3.

4 SAMPLING WITH ACCURACY BOUNDS

In this section, we first propose the sampling method by adapting the random walk and breadth-first search strategies (Section 4.1). We then establish the accuracy bounds of the method (Section 4.2).

4.1 Sampling Strategy

We start with a single-round sampling method.

Single-round sampling. We present two sampling strategies, random walk and uniform sampling with breadth-first search (BFS). Here the BFS strategy makes sure that rules with multiple relational atoms can be discovered with a theoretical guarantee.

Random walk strategy. One might want to uniformly sample tuples from $\mathcal{D}$. However, some predicates (*e.g.,* $t.A \neq s.B$) can be satisfied by a large number of tuples and thus, have low selectivity. Hence, uniform sampling from $\mathcal{D}$ might cause bias towards low selectivity predicates while those rules with high selectivity predicates (*e.g.,* equality and ML predicates) are hard to be discovered. In light of this, we transform the problem of sampling in $\mathcal{D}$ to sampling in a graph G, in which high selectivity predicates are explicitly considered.

We construct a graph $G = (V, E)$ for $\mathcal{D}$, where V is the set of vertices, each of which corresponds to a tuple in $\mathcal{D}$, and E is the set of edges, such that if $e = \langle v_1, v_2 \rangle$ is an edge in E, the tuple pair corresponding to $\langle v_1, v_2 \rangle$ satisfies at least one equality or ML predicate. For simplicity, we use "vertex" and "tuple" interchangeably. Given the maximum length len, we can adopt the random walk algorithm [38] to sample random walks in G with length at most len. However, since $\mathcal{D}$ can be large, constructing G could be time-consuming. Motivated by this, we adopt an online sampling strategy, RandomWalkSampling, to pick sample $\mathcal{D}_s$ from $\mathcal{D}$.

Specifically, we first construct position list indexes (PLI) [53], which groups tuples by attribute values so that we can easily retrieve tuple pairs that satisfy a certain predicate and thus, are connected by an edge in G. For ML predicates p, we adopt the blocking and matching algorithm of [22] to save the satisfied tuple pairs in an auxiliary structure $\mathcal{L}_{\mathsf{ML}}$, such that $\langle t, s \rangle \models p$ for $t \in \mathcal{L}_{\mathsf{ML}}$ and $s \in \mathcal{L}_{\mathsf{ML}}[t]$. We then iteratively sample tuples from $\mathcal{D}$ via random walk. We first draw an initial vertex t based on uniform sampling, such that all tuples in $\mathcal{D}$ are sampled with the same probability. Then, we iteratively sample more tuples by simulating a random walk with length at most len, starting from t: at each step, the random walk either terminates with probability ϵ, or moves to a neighbor of the current vertex with $(1 - \epsilon)$ probability. Here we utilize PLI to determine the neighbors of a given vertex. All vertices in the walk are included in the sample $\mathcal{D}_s$. The entire process iterates until $|\mathcal{D}_s|$ exceeds the maximum sample size.

Example 4: *Consider applying our random walk method to Tables 1 and 2. Suppose that t_4 is first sampled. Then t_3 is the only tuple to sample next, since it satisfies at least one predicate with t_1,* e.g., *$\langle t_1, t_3 \rangle \models p$ when p is t.*zipcode = *s.*zipcode*. In contrast, if the conventional random walk method is adopted, it may end up with $\mathcal{D}_s = \{t_1, t_4\}$, on which no equality or ML predicates hold; as a consequence, we cannot find useful rules such as* $\mathcal{M}_{\mathsf{Bert}}(t_0.\text{org\_name}, t_1.\text{org\_name}) \rightarrow t_0.\text{zipcode} = t_1.\text{zipcode}$.

BFS strategy. Recall that the support of a rule $\varphi = X \rightarrow p_0$ is computed as the distinct number of tuple pairs satisfying RHS predicate p_0. We first uniformly sample a tuple pair $\langle t_0, t_1 \rangle \in \mathcal{D} \times \mathcal{D}$ and then adopt breadth-first search to fetch the (len − 1)-hop neighborhood of t_0 and t_1, denoted as Neigh(t_0, t_1), which includes t_0 and t_1. In other word, if $\langle t_0, t_1 \rangle$ contributes to REEs φ of len tuple variables, BFS ensures that at least one valuation of φ is in Neigh(t_0, t_1). We refer to the BFS search strategy as BFSSampling.

Multi-round sampling. Single-round sampling suffers from the drawback of poor recall, *i.e.,* a rule that holds globally on $\mathcal{D}$ might not be discovered from the small sample $\mathcal{D}_s$. Therefore, we improve the recall by adopting a multi-round sampling strategy.

Specifically, we run RandomWalkSampling or BFSSampling k times, and obtain k samples, namely $\mathcal{D}_s^1, \ldots, \mathcal{D}_s^k$. For each sample $\mathcal{D}_s^i$, we mine a set Σ_s^i of minimal REEs, and finally, we have $\Sigma_s = \bigcup_{i=1}^k \Sigma_s^i$. As will be seen, in Section 4.2, multi-round sampling improves the recall of a fixed size sample from 78% to 89%.

4.2 Theoretical Analysis

We next prove an accuracy bounds for recall based on multi-round sampling with the BFS strategy. Observe that we consider the recall computed by Σ_s and Σ that are discovered by the same discovery algorithm (Section 3.1). We do not consider the precision because the validation of rules in Σ_s could be efficiently evaluated in $\mathcal{D}$ [22] in parallel and invalid rules from Σ_s are removed.

Assumption. We consider datasets that have a power-law degree distribution [49], *i.e.,* a small number of tuples have much higher degree than the others, and a small number of tuple pairs satisfy most REEs. By the definition of support and the BFS strategy, we say that a tuple pair $\langle t_0, t_1 \rangle \models p_0$ *contributes to* the support of a rule $\varphi = X \rightarrow p_0$ if at least one valuation of φ is in $\mathsf{Neigh}(t_0, t_1)$. Thus, we focus on deducing whether $\langle t_0, t_1 \rangle$ that contributes to a rule $\varphi \in \Sigma$ is sampled. Similar to [16], we adopt two parameters ρ_{min} and ρ_{max} to model the power-law distribution *w.r.t.* tuple pairs and REEs, where ρ_{max} (resp. ρ_{min}) is the maximum (resp. minimum) ratio of tuple pairs in $\mathcal{D} \times \mathcal{D}$ that contribute to the support of at least $\beta \times |\Sigma|$ REEs from $\mathcal{D}$. Here β is estimated as $(\frac{\rho_{\mathsf{max}}}{\rho_{\mathsf{min}}})^{-\Delta}$ [16], where Δ is determined by estimation methods of power-law distribution.

Theorem 1: *Given a sampling ratio r, a recall bound β, a support bound σ, a confidence bound δ, and an error constant $\epsilon \in (0, 1)$, multi-round sampling with the* BFS *strategy draws a set $\mathcal{D}_s$ of $k = \lceil \mathsf{ln}\epsilon / \mathsf{ln}\left(1 - \mathsf{exp}(-\frac{\beta^{1-1/\Delta}(\sigma' - \sigma r^2 \beta^{1/\Delta})^2}{3\sigma r^2})\right) \rceil$ with probability $1 - \epsilon$ s.t.*

(1) $\mathsf{recall}(\Sigma, \Sigma_s) \geq \beta$*, where Σ and Σ_s are the set of minimal rules on $\mathcal{D}$ and $\mathcal{D}_s$, respectively; and (2) such* REEs *in Σ_s can be discovered by setting the support bound on $\mathcal{D}_s$ as $\sigma' = \lceil \sigma r^2 \beta^{1/\Delta} + 1 \rceil$.*

Proof sketch. The proof has two parts, for verifying the number of samples in $\mathcal{D}_s$ and the support bound on $\mathcal{D}_s$. We assume the samples in $\mathcal{D}_s$ have maximum size $|\mathcal{D}|r$. The proof is a little involved and is deferred to the full version due to the space constraint.

(1) Number of samples. We set X_i as the percentage of rules φ in Σ such that φ has a valuation in $\mathsf{Neigh}(t_0^{(i)}, t_1^{(i)})$, and $\langle t_0^{(i)}, t_1^{(i)} \rangle$ is the i-th sample from $\mathcal{D} \times \mathcal{D}$. Then $X = \sum_{i=1}^{|\mathcal{D}_s|^2} X_i$ is the percentage of REEs in Σ that can be mined in $\mathcal{D}_s$. Moreover, the expected value $\mathbb{E}[X]$ is bounded by $\frac{|\mathcal{D}_s|^2}{|\Sigma|} \cdot \frac{\beta|\Sigma|\sigma}{\rho_{\mathsf{max}}|\mathcal{D}|^2} \cdot \rho_{\mathsf{min}} = r^2 \sigma \beta^{1+1/\Delta}$ [16]. Using the Chernoff bound $\mathbf{Pr}[X \geq (1 + \epsilon_x)\mathbb{E}[X]] \leq \mathsf{exp}(-\frac{\epsilon_x^2 \mathbb{E}[X]}{3})$ and setting the support in $\mathcal{D}_s$ as $\lceil \sigma r^2 \beta^{1/\Delta} + 1 \rceil$, we get the probability p_{one} of the event that recall is no smaller than β in one sample. If we discover rules across k samples, we have $1 - (1 - p_{\mathsf{one}})^k \geq 1 - \epsilon$. Finally we have that $k = \lceil \mathsf{ln}\epsilon / \mathsf{ln}\left(1 - \mathsf{exp}(-\frac{\beta^{1-1/\Delta}(\sigma' - \sigma r^2 \beta^{1/\Delta})^2}{3\sigma r^2})\right) \rceil$.

(2) Support bound. As in (1), we set the support bound on $\mathcal{D}_s$ as the minimal support within the error parameter ϵ in the sampling domain, by using ϵ, sampling ratio r and the support bound σ. □

Confidence bound. We set the confidence bound on $\mathcal{D}_s$ as $\delta' = \frac{1-\epsilon/4}{1+\epsilon/4}\delta$. Following [7], we set X as the number of valuations satisfied by a rule φ, and also use the Chernoff bound. We assume the same error constant ϵ. The confidence bound on $\mathcal{D}_s$ is deduced as the probability that the preconditions of a rule meet the minimal support σ and the rule with the maximal support [7].

Example 5: *Consider sampling from a database $\mathcal{D}$ of schemas Org and Pers (Example 1) by setting* len, Δ *and* r *as* 4, 0.2 *and* 0.1, *respectively. To get* $\mathsf{recall}(\Sigma, \Sigma_s) \geq 0.9$, *we draw 6 samples via* BFS, *with support* 100, *confidence* 0.9 *and error rate* 0.1 *by Theorem 1. We can discover* REEs *such as φ_2 (see Example 1) from the samples when the support bound and the confidence bound on $\mathcal{D}_s$ are* $\lceil 0.1^2 \cdot 100 \cdot 0.9^{1/0.2} + 1 \rceil = 2$ *and* $\frac{1-0.1/4}{1+0.1/4} \cdot 0.9 = 0.86$, *respectively.*

Remark. The bounds above differ from the prior results for sampling as follows. (1) Taking sampling ratio as input, we provide the first bound on the number of samples with a multi-round sampling strategy, while guaranteeing recall as opposed to an error rate. (2) The number of samples is independent of the size of the dataset $\mathcal{D}$. Apart from the given thresholds and error rate, the bound is determined by knowing only the sampling ratio and support threshold. (3) The Chernoff bounds are extended to rules with relation atoms across multiple tables, and the lower bound of k is independent of the number of multiple tuple pairs that satisfy each rule in Σ_s. (4) We establish the first connections between support/confidence on samples $\mathcal{D}_s$ and their counterparts on the entire dataset $\mathcal{D}$.

5 PARALLEL RULE DISCOVERY

When sampling alone does not suffice to scale with big datasets, we develop a parallel rule discovery algorithm with performance guarantees, namely, *parallel scalability.* Below we first review the parallel scalability (Section 5.1) and present a sequential discovery algorithm RMiner (Section 5.2). We then show how we handle multiple tuple variables in collective rules and how we retrieve constant patterns (Section 5.3). Finally, we provide a parallel algorithm PRMiner that is parallelly scalable relative to RMiner (Section 5.4).

We run the discovery algorithms k times on a set $\{\mathcal{D}_s^1, \ldots, \mathcal{D}_s^k\}$ of samples that are extracted from dataset $\mathcal{D}$ by following the multi-round sampling strategy (Section 4); samples in $\mathcal{D}_s$ have a size bounded by sampling ratio r and guarantee the bounds on precision and recall as stated in Theorem 1, subject to error ratio ϵ. For simplicity, we still use $\mathcal{D}_s$ to denote one sample.

5.1 Parallel Scalability

We revisit the widely adopted notion of parallel scalability [36]. Assume that $\mathcal{A}$ is a sequential algorithm which, given a dataset $\mathcal{D}_s$, consequences RHS, and thresholds σ and δ for support and confidence, respectively, computes a cover Σ_c of minimal σ-frequent and δ-confident REEs on $\mathcal{D}_s$ such that $\mathsf{precision}(\Sigma, \Sigma_s) \geq \alpha$, $\mathsf{recall}(\Sigma, \Sigma_s) \geq \beta$, and the REEs are relevant to consequences RHS. Denote its worst running time as $t(|\mathcal{D}_s|, |\mathsf{RHS}|, \sigma, \delta)$.

Definition 5.1: *A parallel* REE *discovery algorithm* $\mathcal{A}_p$ *is* parallelly scalable relative to $\mathcal{A}$ *if its running time by using n processors is:*

$$T(|\mathcal{D}_s|, |\mathsf{RHS}|, \sigma, \delta,) = \widetilde{O}(\frac{t(|\mathcal{D}_s|, |\mathsf{RHS}|, \sigma, \delta)}{n}),$$

where the notation $\widetilde{O}()$ *hides* $\log(n)$ *factors.*

Intuitively, parallel scalability guarantees "linear" speedup of $\mathcal{A}_p$ relative to the yardstick algorithm $\mathcal{A}$. That is, the more processors are used, the faster $\mathcal{A}_p$ is. Hence $\mathcal{A}_p$ scales well with large databases by adding processors, and makes REE discovery feasible in practice.

5.2 Sequential Algorithm

We start with a sequential rule discovery algorithm, referred to as RMiner, on the sample $\mathcal{D}_s$. As shown in Figure 1, RMiner is a *levelwise search* algorithm. Given a consequence predicate p_0 in RHS and the set $\mathcal{P}_0$ of its correlated predicates, we maintain two predicate sets for discovering new REEs $\varphi : X \rightarrow p_0$ with $X \subseteq \mathcal{P}_0$:

- $\mathcal{P}_{\mathsf{sel}}$, the set of predicates selected to constitute X; and
- $\mathcal{P}_{\mathsf{re}}$, the set of remaining predicates in $\mathcal{P}_0$.

Initially, $\mathcal{P}_{\mathsf{sel}}$ is empty and $\mathcal{P}_{\mathsf{re}}$ is $\mathcal{P}_0$ (line 4). RMiner then traverses the search space level by level by maintaining a queue Q (line 8), where at the i-th level, it discovers candidate REE $\varphi : X \rightarrow p_0$ with $|X| = i$. It iteratively moves predicates from $\mathcal{P}_{\mathsf{re}}$ to $\mathcal{P}_{\mathsf{sel}}$ (lines 15-16) until one of the following conditions is satisfied: (1) $\mathcal{P}_{\mathsf{re}}$ is exhaustive; or (2) $\varphi : \mathcal{P}_{\mathsf{sel}} \rightarrow p_0$ is a minimal REE (σ-frequent and δ-confident; lines 11-13), since in this case, adding more predicates will not make $\mathsf{supp}(\varphi, \mathcal{D})$ larger, while it increases the order of φ. If $\varphi : \mathcal{P}_{\mathsf{sel}} \rightarrow p_0$ is still not a minimal REE, we expand it; before expansion, anti-monotonicity is applied to check whether we can terminate the expansion early (line 14). Finally, the cover of discovered rules is computed and returned (line 6).

Algorithm RMiner employs two optimization strategies commonly used in rule discovery. (a) When multiple p_0's in RHS share similar correlated predicates $\mathcal{P}_0$, it processes these p_0's together (not shown). (b) It pre-computes auxiliary structures (line 2) such as PLI [53], to efficiently compute supports and confidences when verifying whether the mined REEs are above the required thresholds.

5.3 Optimization Strategies

To speed up rule discovery, we propose two optimization strategies, namely, dynamic predicate expansion and constant pattern recovery. The former allows us to discover collective rules across multiple tables efficiently, and the latter complements the sampling strategy, which inevitably drops constants from the dataset.

Dynamical predicate expansion. RMiner might examine all predicate combinations (lines 15-16), which is clearly inefficient. To reduce the enumeration cost, we propose to employ ML models in rule expansion so that we only focus rule discovery among semantically correlated predicates. In the following, we first train an ML model for capturing predicate correlation, and then show how we embed the proposed model in rule discovery.

Learning predicate correlation. Recall that we maintain a set $\mathcal{P}_{\mathsf{sel}}$ of selected predicates, and we iteratively add predicate p to $\mathcal{P}_{\mathsf{sel}}$ and check whether $\mathcal{P}_{\mathsf{sel}} \cup \{p\} \rightarrow p_0$ is a minimal REE. Instead of trying all possible p, we add p to $\mathcal{P}_{\mathsf{sel}}$ only if p and $\mathcal{P}_{\mathsf{sel}}$ are *correlated* for p_0. This correlation, denoted by $\mathcal{M}_{\mathsf{Corr}}(\mathcal{P}_{\mathsf{sel}}, p, p_0)$, is learned via *reinforcement learning (RL)*, and is trained based on the support

```
Algorithm RMiner
Input: $\mathcal{D}_s$, RHS, $\sigma$ and $\delta$.
Output: A cover $\Sigma_s$ of minimal REEs such that for each $\varphi : X \rightarrow p_0$ in $\Sigma$,
  (1) $p_0 \in$ RHS; (2) $X \subseteq \mathcal{P}_0$, where $\mathcal{P}_0$ is a set of predicates correlated to $p_0$.
1. $\Sigma := \emptyset$;
2. Build auxiliary structures, e.g., position list indexes (PLI) [53];
3. for each $p_0 \in$ RHS do
4.     $\mathcal{P}_{sel} := \emptyset$; $\mathcal{P}_{re} := \mathcal{P}_0$;
5.     $\Sigma :=$ Expand($\mathcal{D}_s, \mathcal{P}_{sel}, \mathcal{P}_{re}, p_0, \delta, \sigma, \Sigma$);
6. $\Sigma_s :=$ computeCover($\Sigma$);
7. return $\Sigma_s$;

Procedure Expand
Input: $\mathcal{D}_s$, $\mathcal{P}_{sel}$, $\mathcal{P}_{re}$, $p_0$, $\delta$, $\sigma$ and the current set $\Sigma$ of minimal REEs.
Output: An updated set $\Sigma$ of minimal REEs.
8. $Q$ := an empty queue; $Q$.add($\langle \mathcal{P}_{sel}, \mathcal{P}_{re} \rangle$);
9. while $Q \neq \emptyset$ do
10.     $\langle \mathcal{P}_{sel}, \mathcal{P}_{re} \rangle := Q$.pop(); $\varphi := \mathcal{P}_{sel} \rightarrow p_0$;
11.     if $\varphi$ is minimal then
12.         $\Sigma := \Sigma \cup \{\varphi\}$;
13.         continue; // do not further expand
14.     if supp($\varphi$) $\geq \sigma$ then  // Anti-monotonicity
15.         for each $p \in \mathcal{P}_{re}$ do // Add predicates from $\mathcal{P}_{re}$ to $\mathcal{P}_{sel}$
16.             $Q$.add($\langle \mathcal{P}_{sel} \cup \{p\}, \mathcal{P}_{re} \setminus \{p\} \rangle$);
17. return $\Sigma$;
```

Figure 1: Algorithm RMiner

$\mathsf{supp}(\mathcal{P}_{\mathsf{sel}} \cup \{p\})$ and confidence $\mathsf{conf}(\mathcal{P}_{\mathsf{sel}} \cup \{p\} \rightarrow p_0)$.

Ideally, given a set $\mathcal{P}_{\mathsf{add}}$ of predicates to be added, if $\mathsf{supp}(\mathcal{P}_{\mathsf{sel}} \cup \mathcal{P}_{\mathsf{add}}) \geq \sigma$ and $\mathsf{conf}(\mathcal{P}_{\mathsf{sel}} \cup \mathcal{P}_{\mathsf{add}} \rightarrow p_0) \geq \delta$, then for each p in $\mathcal{P}_{\mathsf{add}}$, $\mathcal{M}_{\mathsf{Corr}}(\mathcal{P}_{\mathsf{sel}}, p, p_0)$ returns true and p is promising and can be added to $\mathcal{P}_{\mathsf{sel}}$; otherwise $\mathcal{M}_{\mathsf{Corr}}(\mathcal{P}_{\mathsf{sel}}, p, p_0)$ returns false and we do not consider p for $\mathcal{P}_{\mathsf{sel}}$. We adopt a classification model, *e.g., feedforward neural network (*FNN*)* for $\mathcal{M}_{\mathsf{Corr}}$, which however, is nontrivial to train due to the exponential cost to generate all labels.

In light of this, we adopt Deep Q-learning (DQN) [48] to generate training instances for $\mathcal{M}_{\mathsf{Corr}}(\mathcal{P}_{\mathsf{sel}}, p, p_0)$; we treat the currently selected $\mathcal{P}_{\mathsf{sel}}$ as state s, the next predicate p to be added as action a, and $\mathsf{conf}(\mathcal{P}_{\mathsf{sel}} \cup \{p\} \rightarrow p_0) - \mathsf{conf}(\mathcal{P}_{\mathsf{sel}} \rightarrow p_0)$ as reward r in DQN. If $\mathsf{supp}(\mathcal{P}_{\mathsf{sel}}) < \sigma$ or $|P_{\mathsf{sel}} \cup \{p\}|$ exceeds the maximum length, we set a very small negative reward, indicating that the predicate combination is not promising. Intuitively, if $\mathcal{P}_{\mathsf{sel}}$ and p are correlated for p_0, the cumulative reward is positive, Otherwise, it is negative. Initially, the first state, denoted by s_1, is an empty $\mathcal{P}_{\mathsf{sel}}$. At the i-th state s_i, DQN determines the next action p to be applied, adds p to $\mathcal{P}_{\mathsf{sel}}$, and transforms s_i to s_{i+1}, denoted by $s_i \rightarrow_p s_{i+1}$.

To determine the next action (e.g., from s_i to s_{i+1}), DQN utilizes two networks: a Q-network and a target network. (1) The Q-network is implemented as a feedforward network with two hidden layers. It takes a state s_i and an action a as inputs, and outputs an estimated value $\hat{Q}$, as the estimated reward of taking action a. It is learned and updated in each state, *e.g.,* from s_i to s_{i+1}. The larger $\hat{Q}$ is, the more likely a is applied. (2) For the target network, its parameters are only updated by the parameters of Q-network in the last state, *e.g.,* s_i. Specifically, it is obtained by cloning Q-network in every state to generate the Q-learning targets for the next update. The Q-network gradually learns its parameters with increasing size of $\mathcal{P}_{\mathsf{sel}}$. Let $\mathcal{P}^i_{\mathsf{sel}}$ be a set of predicates of size i. Given an empty

```
Algorithm LearnRL
Input: $\mathcal{D}_s, \mathcal{P}_0, \sigma, \delta, N$.
Output: $\mathcal{M}_{\text{Corr}}$.
1. Initialize the DQN model $\mathcal{M}$, and the correlation model $\mathcal{M}_{\text{Corr}}$;
2. num := 1;
3. while num $\leq N$ do
4.     Sample $p_0 \in \mathcal{P}_0$;
5.     Train $\mathcal{M}$ by randomly selecting one predicate from $\mathcal{P}_0$ as $\mathcal{P}_{\text{sel}}$
           at a time, and expanding $\mathcal{P}_{\text{sel}}$ until either supp($\mathcal{P}_{\text{sel}}, \mathcal{D}_s$) $< \sigma$ or
           conf($\mathcal{P}_{\text{sel}} \rightarrow p_0, \mathcal{D}_s$) $> \delta$ (controlled by DQN);
6.     Store their supports and confidence in Mem;
7.     num := num + 1;
8. Generate $N$ training instances $\mathcal{T}$ ($\mathcal{P}_{\text{sel}} \cup \{p\} \rightarrow p_0, 0/1$) using $\mathcal{M}$;
9. Train $\mathcal{M}_{\text{Corr}}$ with $\mathcal{T}$;
10. return $\mathcal{M}_{\text{Corr}}$;
```

Figure 2: Algorithm LearnRL

$\mathcal{P}^0_{\text{sel}}$, it is expanded with a certain number ΔL of predicates, one at a time. After that, we obtain a sequence s_{seq} of actions and states, say $\mathcal{P}^0_{\text{sel}}, p_1, \mathcal{P}^1_{\text{sel}}, p_2, \ldots, \mathcal{P}^{\Delta L-1}_{\text{sel}}, p_{\Delta L}, \mathcal{P}^{\Delta L}_{\text{sel}}$, and the value Q^* is:

$$Q^*(s_{\text{seq}}, p) = \mathbb{E}_{s'_{\text{seq}} \sim \xi}[r + \gamma \max_{p'} Q^*(s'_{\text{seq}}, p') | s_{\text{seq}}, p].$$

where ξ is the environment [48], *i.e.,* the rule discovery function, and γ is a discount ratio. In DQN, the approximate value $\hat{Q}$ is learned along with the output $Q^*$of the target network, such that $\hat{Q} \approx Q^*$. Since Q-network takes $\mathcal{P}_{\text{sel}}$ and p as inputs, and outputs a $|\mathcal{P}_0|$-dimensional vector, we transform $\mathcal{P}_{\text{sel}}$ into a $|\mathcal{P}_0|$-dimensional bit vector $\mathbf{v}_{\text{ps}}$,where $\mathbf{v}_{\text{ps}}[p] = 1$ if $p \in \mathcal{P}_{\text{sel}}$, and $\mathbf{v}_{\text{ps}}[p] = 0$ otherwise. The learning method is the same as DQN, and its loss function is:

$$\mathcal{L}(\theta_i) = \mathbb{E}_{s_{\text{seq}}, a \sim \rho(\cdot); s'_{\text{seq}} \sim \xi}[(r + \gamma \max_{a'} \hat{Q}(s'_{\text{seq}}, a'; \theta_{i-1})) - \hat{Q}(s_{\text{seq}}, a; \theta_i)]$$

where $\rho(s_{\text{seq}}, a)$ is the behavior distribution [48], and θ_i denotes parameters of Q-network in the i-th step. Different from DQN, which simply chooses actions from a fixed action set, we select actions, *i.e.,* predicates that do not belong to the current state($\mathcal{P}_{\text{sel}}$). Hence for a given $\mathcal{P}_{\text{sel}}$, when obtaining its corresponding sequence of actions and states, the action set is constantly shrinking. After learning the policy from DQN, we generate a few training instances, e.g., for $\mathcal{P}_{\text{sel}} \cup \{p\}$, we use DQN to select new predicates with the maximum Q values until it is satisfied by p_0 (with label 1) or not (with label 0). Finally we train $\mathcal{M}_{\text{Corr}}$ with these training instances.

Model pre-training. As shown in Figure 2, we pre-train $\mathcal{M}_{\text{Corr}}$ on $\mathcal{D}_s$ using procedure LearnRL. It takes as inputs thresholds σ, δ, a set $\mathcal{P}_0$ of predicates, and the number N of sequences. Here $\mathcal{P}_0$ is constructed on $\mathcal{D}$; we use N to strike a balance between the accuracy of $\mathcal{M}_{\text{Corr}}$ and the training time (by default, $N = 500$; see Section 6). We only pre-train the model once on each dataset.

LearnRL first initializes the DQN model $\mathcal{M}$ and the correlation model $\mathcal{M}_{\text{Corr}}$ (line 1). It then trains $\mathcal{M}$ with N sequences (lines 3-7). For each sequence, we sample a p_0 from $\mathcal{P}_0$ (line 3), and iteratively expand $\mathcal{P}_{\text{sel}}$ (line 5) by adding predicates from $\mathcal{P}_0$, until the support of $\mathcal{P}_{\text{sel}}$ is below σ or the confidence of $\mathcal{P}_{\text{sel}} \rightarrow p_0$ reaches δ. After that, we generate N training instances (line 8) and train $\mathcal{M}_{\text{Corr}}$ to predict whether $\mathcal{P}_{\text{sel}} \cup \{p\} \rightarrow p_0$ is a potential rule or not (line 9). To save time, some training instances for training $\mathcal{M}$ could be reused.

Model fine-tuning. In this stage, we re-generate N' (by default, $N' = 50$) training instances on $\mathcal{D}$ with DQN. Then $\mathcal{M}_{\text{Corr}}$ is fine-tuned and new training instances are also reused in discovery.

After $\mathcal{M}_{\text{Corr}}$ is learned, we make use of it to select predicates, by replacing lines 14-16 of algorithm RMiner with the following:

```
Algorithm CRecover
Input: A template pattern $\varphi_{\text{tem}}$ and the relational instances $\mathcal{D}$.
Output: A set $\Sigma$ of REEs recovered from $\varphi_{\text{tem}}$.
1. Recover non-constant predicates $\mathcal{P}_v$ and $p_0$ from $\varphi_{\text{tem}}$; $\mathcal{P}_c = \emptyset$;
2. for each $p \in \mathcal{P}_v$ do
3.     for each $\langle t, s \rangle$ s.t. $h\langle t, s \rangle \models p$ do
4.         Fill the wildcards in $\varphi_{\text{tem}}$ with the constant values of $t$ and $s$;
5.         Add the recovered constant predicates to $\mathcal{P}_c$;
6. Resume rule discovery and obtain $\Sigma = \{\varphi : X \rightarrow p_0 \mid X \subseteq \mathcal{P}_v \cup \mathcal{P}_c\}$;
7. return $\Sigma$;
```

Figure 3: Algorithm CRecover

```
14. if supp($\varphi$) $\geq \sigma$ then
15.     for each $p \in \mathcal{P}_{\text{re}}$ do // Add predicates from $\mathcal{P}_{\text{re}}$ to $\mathcal{P}_{\text{sel}}$
16.         if $\mathcal{M}_{\text{Corr}}(\mathcal{P}_{\text{sel}}, p, p_0)$ = True then
17.             $Q$.add($\langle \mathcal{P}_{\text{sel}} \cup \{p\}, \mathcal{P}_{\text{re}} \setminus \{p\} \rangle$);
```

That is, we use $\mathcal{M}_{\text{Corr}}$ to prune irrelevant predicates. To avoid pruning true positives, we employ a predefined threshold $\gamma \in [0, 1]$ and adopt the heuristic method to prune only irrelevant predicates inferred by $\mathcal{M}_{\text{Corr}}$ with confidence no small than γ.

Example 6: *For Tables 1 and 2, the set* RHS *collects predicates such as* $p_0 : t_0.\text{org\_name} = t_1.\text{org\_name}$. *After* $\mathcal{M}_{\text{Corr}}$ *is trained, we use it to prune meaningless predicate combinations. For instance,* $\mathcal{M}_{\text{Corr}}(\mathcal{P}_{\text{sel}}, p, p_0)$ = False *for* $\mathcal{P}_{\text{sel}} = \{t_0.\text{zipcode} = t_1.\text{zipcode}\}$ *and* $p : t_0.\text{oid} = t_1.\text{oid}$*; hence* $\mathcal{P}_{\text{sel}}$ *should not be expanded with* p.

We adopt $\mathcal{M}_{\text{Corr}}$ to prune meaningless predicate combinations during discovery. As will be seen in Section 6, $\mathcal{M}_{\text{Corr}}$ substantially reduces the search space and accelerates the discovery process by using DQN. This allows us to discover collective rules that are defined with multiple tuple variables and across different tables. To further accelerate the procedure, we also use heuristic strategies based on confidence [54] to reduce more predicate combinations.

Remark. Note that the training of predicate correlation does not dominate the cost. The computation of predicate correlation mainly includes the model pre-training, label regeneration, and model fine-tuning. During label regeneration, we computes the supports of $\mathcal{P}_{\text{sel}}$ and thus, its results are saved and reused in the discovery step (see Section 5.4). The extra overhead is pre-training and fine-tuning, which in practice, are quite small, since pre-training is done on samples, and the fine-tuning is bounded by $|\mathcal{M}_{\text{Corr}}| \cdot N'$.

Constant pattern. As remarked earlier, sampling inevitably drops constants from $\mathcal{D}$. If we only mine REEs on $\mathcal{D}_s$, we might miss some REEs that hold on $\mathcal{D}$ but not on $\mathcal{D}_s$ due to the absence of some constants from $\mathcal{D}_s$. To compensate the missing constants, we propose a constant pattern recovery strategy. Instead of directly mining constant patterns in $\mathcal{D}_s$, we mine *template patterns* in $\mathcal{D}_s$, which are constant patterns whose constant values are unassigned. Then, we fill in the unassigned value in the template patterns with concrete constants, to recover constant patterns in $\mathcal{D}$.

Template pattern discovery in $\mathcal{D}_s$. We start with the notion of *template patterns*, denoted by $\varphi_{\text{tem}} : (P \rightarrow Q, t_{\text{tem}})$, which are defined analogously to a pattern format REE, except that each constant in the template pattern t_{tem} is replaced with a wildcard '_'. Intuitively, a wildcard can match an arbitrary constant. For example, for the constant pattern at the end of Section 2, we can define a template pattern $\varphi_{\text{tem}} : (P \rightarrow Q, t_{\text{tem}})$, where P, Q remain the same, and

$$t_{\text{tem}} = (\oplus_1.\text{left}, \oplus_2.\text{left}, \_, \oplus_1.\text{right}, \oplus_2.\text{right}, \bar{\_}, \neq, \mathcal{M}||\_).$$

Then instead of directly mining constant patterns in $\mathcal{D}_s$, we mine template patterns in $\mathcal{D}_s$, by revising RMiner as follows. (1) Instead of enumerating multiple constant predicates $t.A \oplus c_1, \ldots, t.A \oplus c_l$ for the same attribute $t.A$, we only identify a *template predicate* $t.A \oplus \_$. (2) We then use template predicates to check which data satisfies them. Given a template predicate $p : t.A \oplus \_$, a valuation h is said to *satisfy* p, written as $h \models p$, if $h(t).A$ exists, regardless of its value; similarly for $h \models \varphi_{\text{tem}}$. (3) Instead of checking the supports and confidences in REEs verification, we say that a template pattern φ_{tem} is *valid* if there exists at least one valuation h such that $h \models \varphi_{\text{tem}}$. Only valid template patterns will be used later for constant pattern recovery. (4) To reduce the complexity of template predicate enumeration, we pre-process $\mathcal{D}_s$ by mining free itemsets [19] from it. For each itemset mined, *e.g.,* $\{t.A = c_1, t.B = c_2\}$, we construct a corresponding *template itemset*, *e.g.,* $\{t.A = \_, t.B = \_\}$. Then, instead of enumerating all combinations of template predicates, we only enumerate template itemsets. This strategy effectively reduces the useless constant combinations that do not exist in $\mathcal{D}_s$, speeding up the template pattern discovery process.

Constant pattern recovery in $\mathcal{D}$. Given $\varphi_{\text{tem}} : (P \rightarrow Q, t_{\text{tem}})$ discovered from $\mathcal{D}_s$, we recover the constant patterns that hold on $\mathcal{D}$, by instantiating the wildcards in φ_{tem} with concrete constants.

To do this, a straightforward way is to retrieve *all* constant values in $\mathcal{D}$, fill them into φ_{tem}, and check the validity. Clearly, this strategy is costly since most tuple pairs in $\mathcal{D}$ may not satisfy non-constant predicates (*e.g.,* $t.A \oplus s.B$) defined in φ_{tem}, and thus, it is unnecessary to try those constants. Besides, directly filling the wildcards might not yield minimal REEs and thus, requires us to resume discovery for removing redundant predicates.

We propose a method, denoted as CRecover, to recover constant patterns based on a given template pattern φ_{tem}, as shown in Figure 3. We first recover the set of all non-constant predicates (line 1), denoted by $\mathcal{P}_v$, based on φ_{tem} (see Section 2 for how to recover predicates from pattern format REEs). Then, we find the tuple pairs $\langle t, s\rangle$ in $\mathcal{D}$ that satisfy at least one predicate in $\mathcal{P}_v$, instantiate the wildcards in φ_{tem} with the constant values of t and s, and obtain a set of constant predicates, denoted by $\mathcal{P}_c$ (lines 2-5). That is, for each $t.A \oplus \_$ in φ_{tem}, we create a constant predicate $t.A \oplus c$ in $\mathcal{P}_c$, where c is the A-attribute value of t; similarly for s. Finally, we resume the rule discovery process based on $\mathcal{P}_v$ and $\mathcal{P}_c$, *i.e.,* using RMiner, to find REEs $\varphi : X \rightarrow p_0$, where $X \subseteq \mathcal{P}_v \cup \mathcal{P}_c$ and p_0 is the RHS specified in φ_{tem} (line 6). Different from complete rule discovery, CRecover focuses discovery on the constants of φ_{tem}, without enumerating all constants and predicate combinations.

There might be a small number of missing templates from $\mathcal{D}_s$. Nonetheless, Theorem 1 ensures that our sampling method would not miss too many cases. The more rounds are used, the less are missed. As will be seen in Section 6, despite the missing cases, the recall remains high, and the discovery cost is substantially reduced.

Example 7: *Consider a template pattern* $\varphi_{\text{tem}} : \text{Org}(t_a) \wedge \text{Org}(t_b) \wedge \text{Pers}(t_c) \wedge \mathcal{M}_{\text{Bert}}(t_a.\text{org\_name}, t_b.\text{org\_name}) \wedge t_a.\text{oid} = t_c.\text{pid} \wedge t_c.\text{person\_address} = \_ \rightarrow t_a.\text{zipcode} = \_$ *from Tables 1 and 2. We first fetch valuations satisfying non-constant predicates in* LHS, *denoted as* (t_x, t_y, t_z). *Then we find out valid values of attribute, e.g.,* t_z.person_address = *Guangzhou and* t_x.zipcode = *510375.*

```
Algorithm PRMiner
Input: $\mathcal{D}_s$, RHS, $\sigma$, $\gamma$, $\Delta L$, and fine-tuned ML model $\mathcal{M}_{\text{Corr}}$.
Output: A cover $\Sigma_s$ of minimal REEs.
 /* executed at coordinator $S_c$ */
1. $i := 0$;  $\Sigma_i := \emptyset$;
2. for each $p_0 \in$ RHS do
3.     Construct a work unit $w = \langle \mathcal{P}_{\text{sel}}, \mathcal{P}_{\text{re}}, p_0 \rangle$, where $\mathcal{P}_{\text{sel}} = \emptyset$ and $\mathcal{P}_{\text{re}} = \mathcal{P}_0$;
4. Evenly divide RHS into $n$ partitions, namely $\text{RHS}_1, \ldots, \text{RHS}_n$;
5. Assign workload $\mathcal{W}_j = \{w = \langle \mathcal{P}_{\text{sel}}, \mathcal{P}_{\text{re}}, p_0 \rangle \mid p_0 \in \text{RHS}_j\}$ to worker $P_j$;
6. $S_c$ distributes $\mathcal{M}_{\text{Corr}}$ to $n$ workers;
 /* Fetch data for n workers */
7. for each worker $P_j$ do
8.     Fetch $\mathcal{D}_{\mathcal{W}_j} = \{t \in \mathcal{D} \mid \exists s \in \mathcal{D}, p \in \mathcal{P}_0$ s.t. $h\langle t,s \rangle \models p$ or $h\langle t,s \rangle \models p_0$,
           where $p_0 \in \text{RHS}_j\}$ and build the corresponding auxiliary structures;
 /* executed at coordinator $S_c$ in levels */
9. while there exists unfinished work do   /* superstep i */
       /* run on n workers in parallel*/
10.    for each $P_j$ with non-empty workload $W_j$ do
11.        $P_j$ runs RMiner from the $i$-th to $(i+1)$-th level in parallel;
12.    for each $P_x$ that has finished the assigned workload do
13.        Balance workload between $P_j$ and $P_x$ ($P_j$ is the heaviest worker);
14.    Upon receiving new REEs from workers, $S_c$ updates $\Sigma_i$ to $\Sigma_{i+1}$;
15.    $i := i + 1$;
16. $\Sigma_s :=$ the cover of $\Sigma_i$; // computed in parallel;
17. return $\Sigma_s$;
```
Figure 4: Algorithm PRMiner

5.4 Parallel Algorithm

We next develop a parallel algorithm, denoted by PRMiner, for rule discovery, and show that it is parallelly scalable relative to RMiner.

Setting. PRMiner runs with one coordinator S_c and n workers $P_1, \ldots, P_n$ under the Bulk Synchronous Parallel (BSP) model [68], where the coordinator is responsible for generating and distributing work units, and the workers parallelly discover rules in a levelwise manner. The overall computation is divided into supersteps, where each superstep corresponds to one level in the levelwise search.

Overview. As shown in Figure 4, PRMiner works by employing the pre-trained ML model $\mathcal{M}_{\text{Corr}}$, as follows. The coordinator S_c maintains a set Σ of REEs (line 1). It generates a set of work units and distributes them evenly to n workers (see below; lines 2-5). It also distributes $\mathcal{M}_{\text{Corr}}$ to all workers (line 6). Upon receiving the work units, each worker fetches related data from S_c and builds the auxiliary structures (lines 7-8); then all workers perform rule discovery in parallel by running RMiner locally (lines 9-11). At the end of each superstep, S_c collects the newly discovered rules from each worker (line 14). Moreover, workloads are adjusted and balanced when needed (see below; line 12-13). Finally, the cover of the set of discovered rules is computed in parallel (line 16) and is returned (line 17).

Workload assignment. S_c evenly partitions RHS into n parts: $\text{RHS}_1, \ldots, \text{RHS}_n$. Based on RHS_j, S_c constructs a set $\mathcal{W}_j$ of work units for worker P_j. Each work unit w in $\mathcal{W}_j$ is a triple $\langle \mathcal{P}_{\text{sel}}, \mathcal{P}_{\text{re}}, p_0 \rangle$, where $\mathcal{P}_{\text{sel}}$ is the selected predicates, $\mathcal{P}_{\text{re}}$ is the remaining predicates and p_0 is a consequence in RHS_j. Initially, $\mathcal{P}_{\text{sel}}$ is empty and $\mathcal{P}_{\text{re}}$ is $\mathcal{P}_0$. After receiving $\mathcal{W}_j$, each P_j fetches the data, say $\mathcal{D}_{\mathcal{W}_j}$, from $\mathcal{D}_s$, consisting of tuples and auxiliary structures that satisfy at least one predicate in $\mathcal{P}_0$ or p_0 (line 7); all workers then run RMiner on $\mathcal{D}_{\mathcal{W}_j}$ in parallel. By doing so, tuples that satisfy multiple predicates are fetched once by each worker, reducing the communication cost.

Workload balancing. At each superstep, we split the workload $\mathcal{W}_j$ of the heaviest worker P_j and assign half of it to an idle worker P_x that has finished its work, in the following two cases.

- If there are more than one work unit in $\mathcal{W}_j$, P_j sends half of $\mathcal{W}_j$ along with the corresponding auxiliary structures to P_x.
- When only one heavy work unit $w = \langle \mathcal{P}_{\text{sel}}, \mathcal{P}_{\text{re}}, p_0 \rangle$ remains in P_j, we partition its data $\mathcal{D}_{\mathcal{W}_j}$. Recall that RMiner expands $\mathcal{P}_{\text{sel}}$ with a new $p \in \mathcal{P}_{\text{re}}$ and validates rule $\mathcal{P}_{\text{sel}} \rightarrow p_0$ at each level. We reduce the validation cost since it dominates. First, we select $p' \in \mathcal{P}_{\text{sel}}$ that has the largest support, where p' is $t.A \oplus s.B$ or $\mathcal{M}(t.A, s.B)$. Then we partition attributes $t.A$ and $s.B$ into h parts, and divide the data into $h \times h$ partitions. We send half of these from P_j to P_x along with auxiliary structures of $\mathcal{P}_{\text{sel}} \setminus \{p'\}$ and p_0. Finally, P_x computes the supports and confidences, and sends them back to P_j; P_j integrates the results and returns valid rules at current superstep.

Learning predicate correlation in parallel. We also train the predicate correlation model $\mathcal{M}_{\text{Corr}}$ in parallel. More specifically, similar to algorithm LearnRL (Figure 2), coordinator S_c maintains N sequences, such that N initial $\mathcal{P}_{\text{sel}}$'s are randomly sampled and expanded until their supports are below σ or they make valid rules. Different from the sequential version, we distribute the time-consuming support and confidence computation evenly to all workers. Recall that each sequence contains a set $\mathcal{P}_{\text{sel}}$ and a newly added predicate p. For each valid $\mathcal{P}_{\text{sel}}$ in each sequence such that $\mathcal{P}_{\text{sel}} \rightarrow p_0$ is a candidate rule, S_c employs the current $\mathcal{M}_{\text{Corr}}$ to pick a predicate p with the maximum predicted reward. Then, S_c distributes N such sequences to n workers evenly, such that the supports and confidences of $\mathcal{P}_{\text{sel}} \cup \{p\} \rightarrow p_0$ are computed in parallel. After all workers finish their computations, they send the actual rewards back to S_c, based on which S_c continues to train $\mathcal{M}_{\text{Corr}}$. When N sequence expansions are finalized, the model $\mathcal{M}_{\text{Corr}}$ is learned.

Example 8: *Consider n=3 and N=3. Assume that coordinator S_c randomly selects N valid $\mathcal{P}_{\text{sel}}$'s,* e.g., *$\mathcal{P}_{\text{sel}_1} = \{p_1, p_2, p_6\}$, $\mathcal{P}_{\text{sel}_2} = \{p_2, p_3, p_4\}$ and $\mathcal{P}_{\text{sel}_3} = \{p_4, p_6, p_7\}$ in level 3. It examines predicates p_5, p_1, and p_2 separately with the maximum predicted rewards* w.r.t. *$\mathcal{P}_{\text{sel}_1}$, $\mathcal{P}_{\text{sel}_2}$, and $\mathcal{P}_{\text{sel}_3}$ by current $\mathcal{M}_{\text{Corr}}$. It then distributes $\langle \mathcal{P}_{\text{sel}_1}, p_5 \rangle$, $\langle \mathcal{P}_{\text{sel}_2}, p_1 \rangle$ and $\langle \mathcal{P}_{\text{sel}_3}, p_2 \rangle$ to three workers to compute support and confidence in parallel. The coordinator receives rewards from the workers and trains $\mathcal{M}_{\text{Corr}}$ incrementally. The process iterates until N many $\mathcal{P}_{\text{sel}}$'s have all been expanded.*

Parallel constant recovery. Given a set Σ_{tem} of template patterns, we support parallel constant recovery, denoted by PCRecover, as follows. We divide Σ_{tem} into n partitions, and distribute them to workers. All workers run CRecover in parallel and output the results. If worker P_j has a workload heavier than the others, we handle the skewness using a balancing strategy similar to PRMiner.

Parallel cover computation. Implication checking for REEs can be parallelized in the same way as its counterpart for GFDs, for which a parallelly scalable algorithm is already in place [20]. Our algorithm for computing the cover of discovered REEs is developed along the same lines as [20]. The algorithm retains the parallel scalability.

Parallel scalability. The parallel scalability is shown as follows.

Theorem 2: *Algorithm* PRMiner *(resp.* PCRecover*) is parallelly scalable relative to the sequential algorithm* RMiner *(resp.* CRecover*).*

Proof. For PRMiner, observe that the worst-case time complexity of RMiner is $t(|\mathcal{D}|, |\text{RHS}|, \sigma, \delta, \alpha, \beta) = O(\sum_{\varphi \in C(\mathcal{P}_0) \times \text{RHS}} |\mathcal{D}|^{|\varphi|})$, where $C(\mathcal{P}_0)$ is the power set of $\mathcal{P}_0$ and $|\varphi|$ denotes the number of predicates in φ. In PRMiner, coordinator S_c conducts workload assignment in $O(|\text{RHS}|)$ time. The cost at each worker is dominated by the following: (a) fetch its corresponding data in time at most $O(|\mathcal{D}|)$; (b) transmit the mined rules to the coordinator in at most $O(|\mathcal{D}|)$ time; (c) balance its workload, such that at most $O(|\mathcal{D}|)$ data is sent to idle workers; and (d) locally conduct discovery in $\frac{t(|\mathcal{D}|, |\text{RHS}|, \sigma, \delta, \alpha, \beta)}{n}$ time, since the workload is evenly distributed in (d). Taken together, PRMiner takes at most $\frac{t(|\mathcal{D}|, |\text{RHS}|, \sigma, \delta, \alpha, \beta)}{n}$ time, and is thus parallelly scalable relative to RMiner.

In PCRecover, coordinator S_c splits template patterns in $O(|\Sigma_{\text{tem}}|)$ time, which is much smaller than the cost of CRecover, denoted by *cost*(CRecover). Each worker (1) balances its workload with at most $O(|\mathcal{D}|)$ communication cost; and (2) performs constant recovery in $\frac{O(cost(\text{CRecover}))}{n}$ time since workload is balanced. Thus, PCRecover is parallelly scalable relative to CRecover. $\square$

The cost of training. The learning time of $\mathcal{M}_{\text{Corr}}$ is also much smaller than $O(\frac{t(|\mathcal{D}|, |\text{RHS}|, \sigma, \delta, \alpha, \beta)}{n})$ time. Indeed, $\mathcal{M}_{\text{Corr}}$ is implemented with DQN of two hidden layers. Denote the dimension of each layer as h; then $|\mathcal{M}_{\text{Corr}}|$ is in $O(h(h + |\mathcal{P}_0|))$. In practice, h is relatively small, *e.g.*, $h \leq 10^3$ [48]. As the number of epoch (*i.e.*, iteration) to train $\mathcal{M}_{\text{Corr}}$ is a constant value, the learning time of $\mathcal{M}_{\text{Corr}}$ is bounded by $O(Nh(h + |\mathcal{P}_0|))$. Moreover, the most time-consuming computation of supports and confidences is conducted in parallel. As will be seen in Section 6, the cost is relatively small.

6 EXPERIMENTAL STUDY

We experimentally evaluated (1) the efficiency and accuracy of the proposed sampling strategy, constant pattern recovery and correlated predicate learning, (2) the scalability of PRMiner, and (3) the effectiveness of REE discovery in real-life and synthetic datasets.

Experimental setting. We start with the experimental setting.

Datasets. We used seven datasets, including six real-life datasets and a synthetic one (Table 4). Airport, Hospital, Inspection and NCVoter are commonly used in the existing studies [44, 53]. DBLP is an academic dataset with multiple relations. Realty is a property dataset with 12 relations. We also used a synthetic dataset Tax, modified from the tax data (1M) [6, 12], by duplicating each original tuple 10 times and modifying the attributes using a program of [18].

ML models. REEs use three ML models as predicates: ditto [43] for ER, and Bert [55] and edit distance (ED) for a few textual attributes. For predicate correlation model $\mathcal{M}_{\text{Corr}}$, we use 2 hidden layers and set their dimension to 200. We used Adam optimizer with a batch-size of 64; the learning rate is 0.0001. We trained our model with 300 epochs on all datasets. By default, $N = 500$, $N' = 50$, and the inference of $\mathcal{M}_{\text{Corr}}$ is re-implemented by EJML library [1].

Baselines. We implemented the following, all in Java: (1) PRMiner, including RandomWalkSampling and PCRecover; (2-5) four variants of PRMiner: $\text{PRMiner}_{\text{RS}}$ that replaces RandomWalkSampling with the standard random sampling; $\text{PRMiner}_{\text{full}}$ to discover REEs directly from $\mathcal{D}$; $\text{PRMiner}_{\text{noml}}$ without using dynamic predicate expansion; and $\text{PRMiner}_{\text{noCR}}$ without constant pattern recovery; (6) DCfinder [53], a state-of-the-art algorithm that mines all bi-variable DCs that hold on the dataset; we tested DCfinder due to its superiority over other DC discovery methods, as shown in [53]; we parallelize DCfinder for a fair comparison; and (7) REEFinder, a revision of DCfinder to mine bi-variable REEs in parallel by extending the primitives in [59] and adding new primitives to support constant and ML predicates; REEFinder has the same discovery procedure as DCfinder, except its use of the new/extended primitives for constant and ML predicates.

We compared with the four variants of PRMiner to test the effectiveness of random walk, dynamic predicate expansion and constant pattern recovery, respectively, and with DCfinder for efficiency although DCfinder only mines bi-variable DCs, which are a special case of REEs. For large datasets, *e.g.,* DBLP, NCVoter and Tax, we included 4 predicates in RHS, which is typical in an application. For all algorithms, we extract constants (frequency $\geq 20\%$) from all non-categorical attributes as constant predicates, and only consider equality and ML predicates for simplicity.

We conducted experiments on a cluster of up to 21 virtual machines (one for the coordinator), each powered by 64 GB RAM and 18 processors with 3.10 GHz. We ran the experiments 3 times, and report the average here. We do not include the time of loading datasets, pre-computing results of ML predicates and constructing auxiliary structure, *i.e.,* PLI for all algorithms. Unless stated explicitly, we set $\sigma = 10^{-4}|\mathcal{D}_s|^2$, $\delta = 0.9$, $\alpha = 0.8$, $\beta = 0.9$, and $r = 0.1$. We deduce the number k of samples and thresholds for support and confidence on samples following Theorem 1.

Experimental results. We next report our findings.

Exp-1: Sampling. We first evaluated the usefulness of the sampling strategy, by running PRMiner and its four variants on the entire dataset and on samples $\mathcal{D}_S$. We report the accuracy (precision and recall) of our sampling strategy. We also tested the efficiency of (1) the support threshold σ, (2) the confidence threshold δ, (3) the precision threshold α, (4) the recall threshold β, and (5) the number k of samples. The exact values of recall are computed in our experiments by also discovering the set Σ of REEs from the entire datasets in parallel. We found that recall of RandomWalkSampling is 5% higher than BFSSampling on average. Thus for the lack of space, we only show the results of RandomWalkSampling on some of the datasets; the results on the other datasets are consistent.

Accuracy. Varying sampling ratio r from 0.1 to 0.4 (Figures 5(a) and 5(b)) and the number k of samples from 1 to 8 (Figures 5(c), 5(d) and 5(e)), we first tested the accuracy of the methods on Airports and Inspection. We find the following. (a) On average, PRMiner has precision 90% and recall 81%, when $r = 0.1$ and $k = 3$ on Inspection and Airport. (b) Constant pattern recovery improves the recall by 2% on Airport, as indicated by PRMiner vs. $\text{PRMiner}_{\text{noCR}}$. This justifies the need for PCRecover. (c) The use of $\mathcal{M}_{\text{Corr}}$ speeds up rule discovery (see below) without much reduction in accuracy. The precision and recall of PRMiner are only 1% and 5% lower

Name	Type	#tuples	#attributes	#relations
Airport [44, 53]	real-life	55,113	18	1
Hospital [6, 12, 44, 53]	real-life	114,919	15	1
Inspection [44, 53, 56]	real-life	220,940	17	1
NCVoter [44, 53]	real-life	1,681,617	12	1
DBLP [65]	real-life	1,799,559	18	3
Realty	real-life	642,257	110	12
Tax [6, 12, 18, 44, 53]	synthetic	10,000,000	15	1

Table 4: Dataset statistic

than those of $\text{PRMiner}_{\text{noml}}$ when $r = 0.2$ and $k = 2$, respectively. (d) On average PRMiner outperforms $\text{PRMiner}_{\text{RS}}$ in precision and recall by 2.3% and 7%, respectively; this justifies the effectiveness of RandomWalkSampling. (e) When r increases, both precision and recall improve, as expected, since $\mathcal{D}_s$ retains more data of $\mathcal{D}$. (f) The recall also improves with larger k for the same reasons. However, the precision might fluctuate slightly with larger k (note that the y-axis of Figure 5(d) has increment 0.05). This is because there are more rules pertaining to $\mathcal{D}_s$ but not to $\mathcal{D}$. This said, the overall accuracy (precision and accuracy) is improved when k gets larger.

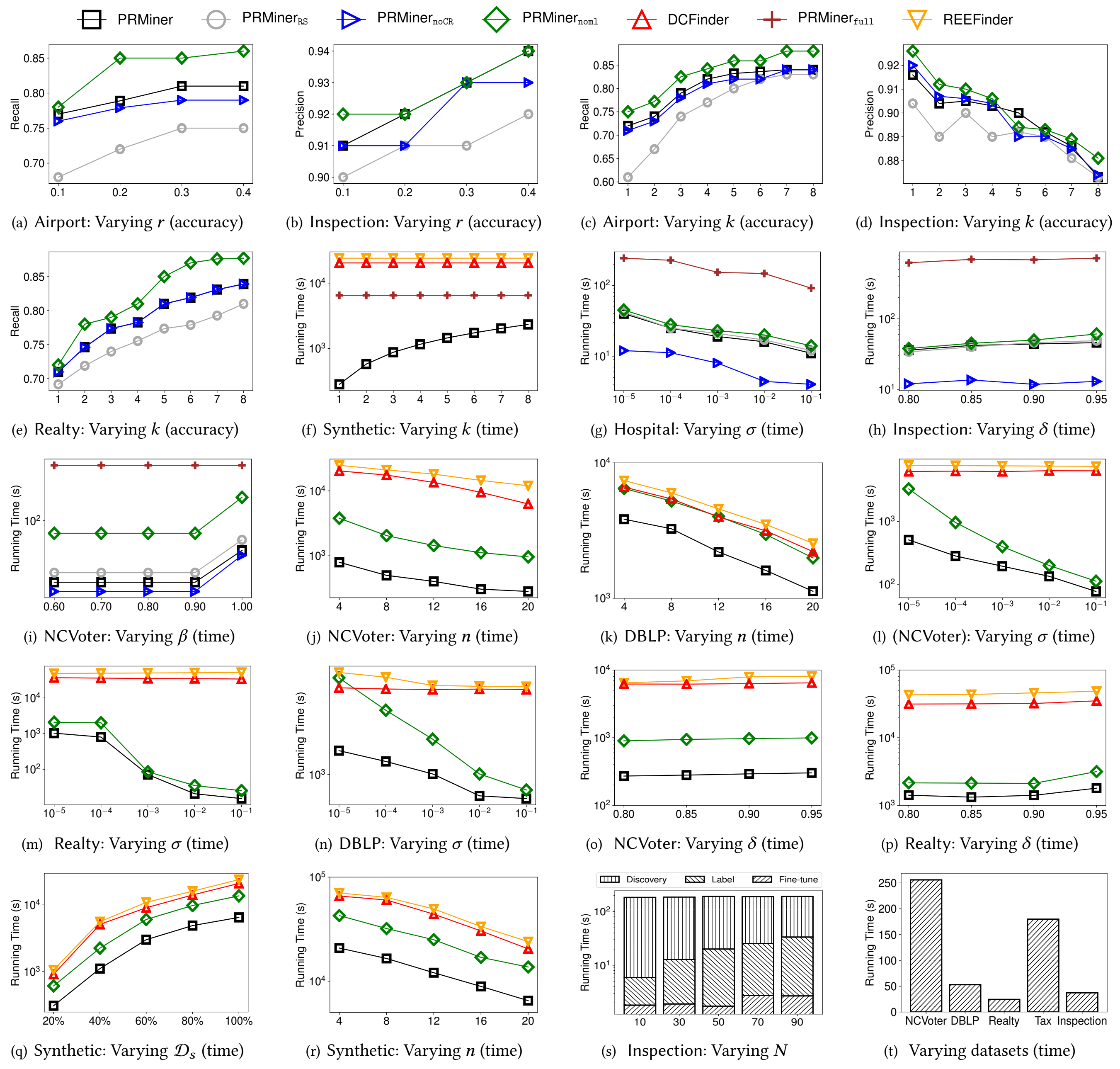
Varying k. We varied k from 1 to 8. As shown in Figure 5(e) over multi-table Realty, recall of PRMiner increases when k gets larger, *e.g.,*0.83 when $k = 7$. As shown in Figure 5(f) over synthetic data, (a) as k increases, the discovery time of PRMiner also increases linearly, as expected. (b) PRMiner with the multi-round sampling is still faster than the discovery in the entire dataset, *e.g.,* when k is as large as 8. This indicates that multi-round sampling supports large value of k and is able to achieve both effectiveness and efficiency.

Varying σ. We then tested PRMiner by varying the support threshold σ from $10^{-5}|\mathcal{D}_s|^2$ to $10^{-1}|\mathcal{D}_s|^2$. As shown in Figure 5(g) on Hospital, (a) it takes PRMiner much less time to run on the sample $\mathcal{D}_s$ than on the entire dataset $\mathcal{D}$, *e.g.,* PRMiner takes 25s on $\mathcal{D}_s$, as opposed to 228s by $\text{PRMiner}_{\text{full}}$ on $\mathcal{D}$ ($\sigma = 10^{-4}|\mathcal{D}_s|^2$). In general, PRMiner is much faster than $\text{PRMiner}_{\text{full}}$, *e.g.,* 9.3 times faster when $\sigma = 0.01|\mathcal{D}_s|^2$. This verifies sampling effectively speeds up the discovery process. (b) When σ is smaller, it takes PRMiner longer since it needs to examine more candidates, *e.g.,* PRMiner is 4.1 times faster when σ varies from $10^{-5}|\mathcal{D}_s|$ to $10^{-1}|\mathcal{D}_s|$. (c) $\text{PRMiner}_{\text{noCR}}$ is faster than PRMiner since it does not recover constant patterns, with the price of lower accuracy as shown above. $\text{PRMiner}_{\text{noml}}$ is slower than PRMiner since it does not use $\mathcal{M}_{\text{Corr}}$ to prune. The effectiveness of $\mathcal{M}_{\text{Corr}}$ is more evident on larger datasets (see Exp-2).

Varying δ. Varying confidence δ from 0.8 to 0.95, we report the results in Figure 5(h) on Inspection. As shown there, PRMiner becomes slightly faster given a smaller δ, *e.g.,* PRMiner is 1.28 times faster when $\delta = 0.8$ than when $\delta = 0.95$. This is because with larger δ, more minimal REEs have to be checked; hence longer time. Also from Figure 5(h), sampling substantially reduces the runtime.

Varying β. We varied the recall threshold β from 0.6 to 1.0. A higher recall comes with the price of a longer execution time. Nevertheless, our sampling method shows convincing reduction on the sample size and the execution time, *e.g.,* when $\beta = 0.6$, it only takes PRMiner 28s on $\mathcal{D}_s$, as opposed to 282s by $\text{PRMiner}_{\text{full}}$ on $\mathcal{D}$.

Exp-2: Scalability We next studied the scalability of PRMiner on (large) entire dataset $\mathcal{D}$, by varying (1) the number n of machines,



(a) Airport: Varying r (accuracy) (b) Inspection: Varying r (accuracy) (c) Airport: Varying k (accuracy) (d) Inspection: Varying k (accuracy)

(e) Realty: Varying k (accuracy) (f) Synthetic: Varying k (time) (g) Hospital: Varying σ (time) (h) Inspection: Varying δ (time)

(i) NCVoter: Varying β (time) (j) NCVoter: Varying n (time) (k) DBLP: Varying n (time) (l) (NCVoter): Varying σ (time)

(m) Realty: Varying σ (time) (n) DBLP: Varying σ (time) (o) NCVoter: Varying δ (time) (p) Realty: Varying δ (time)

(q) Synthetic: Varying $\mathcal{D}_s$ (time) (r) Synthetic: Varying n (time) (s) Inspection: Varying N (t) Varying datasets (time)

Figure 5: Performance evaluation

(2) the support threshold σ, (3) the confidence threshold δ, and (4) the size of synthetic data. Unless stated explicitly, we set $n = 20$, $\sigma = 10^{-4}|\mathcal{D}|^2$, $\delta = 0.9$ by default. The results when varying β are consistent with their counterparts in Exp-1 and hence not shown.

Varying n. We first varied the number n of machines from 4 to 20. As shown in Figures 5(j) and 5(k) on NCVoter and DBLP, respectively, (a) PRMiner scales well with the increase of machines: it is 3.11 times faster when n varies from 4 to 20. (b) PRMiner is feasible in practice. It takes 286s on NCVoter when $n = 20$, as opposed to 6292s by DCfinder and 8470s by REEFinder. This is because PRMiner utilizes σ for early termination, while the other two compute the evidence set as one part of the discovery methods, which is independent of σ and δ, and dominates their rule-discovery costs. (c) REEFinder is slower than DCfinder since it mines bi-variable REEs with constants and ML predicates, beyond bi-variable DCs targeted by DCfinder. (d) PRMiner is 3.63 times faster than PRMiner$_{\mathrm{noml}}$ on average, up to 4.88 times. This verifies that our dynamic predicate expansion effectively reduces the execution time.

Varying σ. Varying the support threshold σ from $10^{-5}|\mathcal{D}|^2$ to $10^{-1}|\mathcal{D}|^2$, we report the results in Figures 5(l), 5(n) and 5(m) on NCVoter, DBLP and Realty, respectively. PRMiner and PRMiner$_{\mathrm{noml}}$ take longer when σ is smaller, as expected, which is consistent with the results in Figure 5(g); DCfinder and REEFinder are not very sensitive to σ as explained above. This said, PRMiner is faster than $\text{PRMiner}_{\text{noml}}$, REEFinder and DCfinder under all values of σ, *e.g.,* PRMiner is 2.97, 27.19 and 19.38 times faster than them on NCVoter on average, respectively, when $\sigma = 10^{-5}|\mathcal{D}|^2$. It also performs the best over Realty, *e.g.,* 35.89 and 50.44 times faster than DCfinder and REEFinder, respectively; this verifies that our discovery algorithm is robust on datasets with multiple relational tables. In particular, the runtime of PRMiner increases more slightly than $\text{PRMiner}_{\text{noml}}$ when σ decreases, since it checks much less REEs than $\text{PRMiner}_{\text{noml}}$ due to its optimization strategies.

Varying δ. We varied the confidence threshold δ from 0.8 to 0.95. As shown in Figures 5(o) and 5(p), PRMiner is slightly faster in most cases when δ decreases, while the others are not very sensitive. Nonetheless, PRMiner consistently beats all its competitors, *e.g.,* it is on average 1.59 and 22.09 (resp. 3.32 and 21.98) times faster than $\text{PRMiner}_{\text{noml}}$ and DCfinder on Realty (resp. NCVoter), respectively. Again this verifies that PRMiner performs well over multiple tables.

Using large Tax synthetic data (15 attributes and 10M tuples), we tested the impact of the size $|\mathcal{D}|$ and the number n of machines.

Varying $|\mathcal{D}|$ (synthetic). Varying the scaling factor of $\mathcal{D}$ from 20% to 100%, *i.e.,* we changed the tuples per relation from 2 million to 10 million, we computed the corresponding sample $\mathcal{D}_s$ on each $\mathcal{D}$. As shown in Figure 5(q), all algorithms take longer, as expected. However, PRMiner still outperforms $\text{PRMiner}_{\text{noml}}$, REEFinder and DCfinder. It takes 308s when $\mathcal{D}$ has 2M tuples, as opposed to 610s, 1059s and 912s by the other three, respectively.

Varying n (synthetic). Fixing the data size $|\mathcal{D}|$ as 10M, we varied the number n of machines from 4 to 20. The results are reported in Figure 5(r) and are consistent with those in Figures 5(j) and 5(k). PRMiner is 3.38 times faster when n varies from 4 to 20.

Fine-tuning cost. Varying N from 10 to 90, we report the fine-tuning cost in Figure 5(s), which has two parts, *i.e.,* the cost of re-generating labels (*i.e.,* computing supports and confidences) of N sequences in $\mathcal{D}$, and the cost of fine-tuning $\mathcal{M}_{\text{Corr}}$. As shown there, (1) the fine-tuning cost increases as N increases, while the overall time is relatively stable. (2) The training is fast, e.g., 2.69s when $N = 90$. (3) The fine-tuning cost is much smaller than the discovery time.

Pre-training cost. We evaluated the pre-training cost among five large datasets in Figure 5(t). The pre-training cost does not dominate the discovery procedure, *e.g.,* the longest time is 256s on NCVoter. Considering the speedup of PMiner against $\text{PRMiner}_{\text{noml}}$, the pre-training step is very useful for accelerating the discovery.

Exp-3: Effectiveness. We manually examined REEs discovered by PRMiner from samples of DBLP and Airport. Below are examples.

(1) ψ_1: $\text{Paper}_{\text{DBLP}}(t_0) \wedge \text{Paper}_{\text{DBLP}}(t_1) \wedge \mathcal{M}_{\text{ED}}(t_0.\text{title}, t_1.\text{title}) \wedge \mathcal{M}_{\text{Bert}}(t_0.\text{venue}, t_1.\text{venue}) \wedge t_0.\text{year} = t_1.\text{year} \rightarrow t_0.\text{id} = t_1.\text{id}$. This REE is an ER rule. It says that if two papers have similar titles and similar venues, and if they were published in the same year, then the two denote the same paper. It employs ML models, *e.g.,* Bert, to check the semantic similarity of titles and venues, which are not supported by prior data quality rules such as CFDs and DCs.

(2) ψ_2: $\text{Author}_{\text{DBLP}}(t_0) \wedge \text{Author}_{\text{DBLP}}(t_1) \wedge t_0.\text{affiliation} = t_1.\text{affiliation} \wedge \mathcal{M}_{\text{Bert}}(\text{t}_0.\text{name}, \text{t}_1.\text{name}) \rightarrow \mathcal{M}_{\text{ditto}}(t_0.\bar{A}, t_1.\bar{A})$, where $\bar{A}$ is the set of all attributes in relation Author of DBLP. It says that for two authors, it is because their names are similar and their affiliations are the same that the ML model predicts the two to match. It makes an attempt to explain the predication of a black box ML model $\mathcal{M}_{\text{ditto}}$ in terms of logic characteristics.

(3) ψ_3: $\text{Airport}(t_0) \wedge \text{Airport}(t_1) \wedge \text{Airport}(t_2) \wedge t_0.\text{continent} = t_2.\text{continent} \wedge t_0.\text{latitude\_deg} = t_1.\text{latitude\_deg} \wedge t_0.\text{iso\_region} = t_2.\text{iso\_region} \wedge t_0.\text{latitude\_deg} = t_2.\text{latitude\_deg} \wedge t_0.\text{municipality} = t_1\text{municipality} \rightarrow t_1.\text{ios\_region} = t_2.\text{iso\_region}$. This rule involves three tuple variables. It indicates that if two tuples both share a few attributes with the third one, they may have the same ios_region value. In detail, for three airports t_0, t_1, t_2, if t_0 and t_2 have the same iso_region, continent and latitude_deg, and if t_0 and t_1 have the same municipality and latitude_deg, then t_1 and t_2 have the same iso_region. It shows that PRMiner is able to discover rules with multiple tuple variables from sample $\mathcal{D}_S$. No prior algorithms discover rules with more than two tuple variables.

(4) ψ_4: $\text{Airport}(t_0) \wedge \text{Airport}(t_1) \wedge t_0.\text{iso\_country} \neq t_1.\text{iso\_country} \rightarrow t_0.\text{municipality} \neq t_1.\text{municipality}$. It says that two airports have different municipalities if their countries are different. It shows that PRMiner is able to find rules that distinguish entities/attributes. Such rules are needed for, *e.g.,* catching mismatched entities.

Summary. We find the following. (1) When $r = 0.1$ and $k = 2$, PRMiner on samples is 12.2 times faster than $\text{PRMiner}_{\text{full}}$ on the entire datasets, on average over all the datasets tested. In particular, over DBLP that has 3 relations, 18 attributes and 1.8M tuples, PRMiner with sampling takes 406s, as opposed to 2960s and 3148s by $\text{PRMiner}_{\text{full}}$ and DCfinder, respectively, when $n = 20$. (2) With the sample ratio 10%, the precision and recall of PRMiner are 90% and 82% (up to 93% and 85%), respectively, which are 2% and 7% higher than random sampling. (3) PRMiner is parallelly scalable: it is 3.38 times faster when the number n of machines varies from 4 to 20. (4) The proposed optimization methods are effective. Employing dynamic predicate expansion, PRMiner is 2.52 times faster than $\text{PRMiner}_{\text{noml}}$ on average, up to 4.77 times. Constant recovery improves the recall of $\text{PRMiner}_{\text{noCR}}$ by 2%. (5) PRMiner is capable of finding useful REEs from real-life data.

7 CONCLUSION

We have studied parallel rule discovery with sampling. The novelty of this work consists of the following: (1) a multi-round sampling strategy with accuracy guarantees; (2) a rule discovery algorithm with the parallel scalability; (3) an ML-based predicate expansion strategy to efficiently discover collective rules with multiple relation atoms; and (4) a method to efficiently retrieve constant patterns. Our experimental study has verified that the method is promising.

One topic for future work is to tighten the accuracy bounds of sampling. Another topic is to integrate data repairing and rule discovery, to enrich each other and reduce the impact of noise.

ACKNOWLEDGMENTS

This work was supported by ERC 652976, Royal Society Wolfson Research Merit Award WRM/R1/180014, NSFC 61902274 and Longhua Science and Technology Innovation Bureau LHKJCXJCYJ202003.

REFERENCES

[1] [n.d.]. http://ejml.org/wiki/index.php?title=Main_Page

[2] Ziawasch Abedjan, Patrick Schulze, and Felix Naumann. 2014. DFD: Efficient functional dependency discovery. In *CIKM*. 949–958.

[3] Serge Abiteboul, Richard Hull, and Victor Vianu. 1995. *Foundations of Databases*. Addison-Wesley.

[4] Marcelo Arenas, Leopoldo Bertossi, and Jan Chomicki. 1999. Consistent Query Answers in Inconsistent Databases. In *PODS*.

[5] Indrajit Bhattacharya and Lise Getoor. 2007. Collective entity resolution in relational data. *TKDD* (2007).

[6] Tobias Bleifuß, Sebastian Kruse, and Felix Naumann. 2017. Efficient Denial Constraint Discovery with Hydra. *PVLDB* 11, 3 (2017), 311–323.

[7] Venkatesan T. Chakaravarthy, Vinayaka Pandit, and Yogish Sabharwal. 2009. Analysis of sampling techniques for association rule mining. In *ICDT*. 276–283.

[8] B. Chandra and Shalini Bhaskar. 2011. A new approach for generating efficient sample from market basket data. *Expert Syst. Appl.* 38, 3 (2011), 1321–1325.

[9] Bin Chen, Peter J. Haas, and Peter Scheuermann. 2002. A new two-phase sampling based algorithm for discovering association rules. In *SIGKDD*.

[10] Chyouhwa Chen, Shi-Jinn Horng, and Chin-Pin Huang. 2011. Locality sensitive hashing for sampling-based algorithms in association rule mining. *Expert Syst. Appl.* 38, 10 (2011), 12388–12397.

[11] Vassilis Christophides, Vasilis Efthymiou, Themis Palpanas, George Papadakis, and Kostas Stefanidis. 2021. An Overview of End-to-End Entity Resolution for Big Data. *ACM Comput. Surv.* 53, 6 (2021), 127:1–127:42.

[12] Xu Chu, Ihab F. Ilyas, and Paolo Papotti. 2013. Discovering Denial Constraints. *PVLDB* (2013).

[13] Kun-Ta Chuang, Ming-Syan Chen, and Wen-Chieh Yang. 2005. Progressive Sampling for Association Rules Based on Sampling Error Estimation. In *PAKDD*.

[14] E. F. Codd. 1972. Relational Completeness of Data Base Sublanguages. *Database Systems: 65-98, Prentice Hall and IBM Research Report RJ 987* (1972).

[15] Jacob Devlin, Ming-Wei Chang, Kenton Lee, and Kristina Toutanova. 2019. BERT: Pre-training of Deep Bidirectional Transformers for Language Understanding. In *NAACL-HLT*. 4171–4186.

[16] Wenfei Fan, Wenzhi Fu, Ruochun Jin, Ling Lu, and Chao Tian. 2022. Discovering Association Rules from Big Graphs. In *VLDB*.

[17] Wenfei Fan, Hong Gao, Xibei Jia, Jianzhong Li, and Shuai Ma. 2011. Dynamic constraints for record matching. *VLDB J.* 20, 4 (2011), 495–520.

[18] Wenfei Fan, Floris Geerts, Xibei Jia, and Anastasios Kementsietsidis. 2008. Conditional functional dependencies for capturing data inconsistencies. *TODS* 33, 2 (2008), 6:1–6:48.

[19] Wenfei Fan, Floris Geerts, Jianzhong Li, and Ming Xiong. 2011. Discovering conditional functional dependencies. *TKDE* 23, 5 (2011), 683–698.

[20] Wenfei Fan, Xueli Liu, and Yingjie Cao. 2018. Parallel Reasoning of Graph Functional Dependencies. In *ICDE*. 593–604.

[21] Wenfei Fan, Ping Lu, and Chao Tian. 2020. Unifying Logic Rules and Machine Learning for Entity Enhancing. *Science China Information Sciences* (2020).

[22] Wenfei Fan, Chao Tian, Yanghao Wang, and Qiang Yin. 2021. Discrepancy Detection and Incremental Detection. *PVLDB* (2021).

[23] Peter A Flach and Iztok Savnik. 1999. Database dependency discovery: A machine learning approach. *AI communications* 12, 3 (1999), 139–160.

[24] Eve Garnaud, Nicolas Hanusse, Sofian Maabout, and Noël Novelli. 2014. Parallel mining of dependencies. In *HPCS*. IEEE, 491–498.

[25] Chang Ge, Ihab F. Ilyas, and Florian Kerschbaum. 2019. Secure Multi-Party Functional Dependency Discovery. *PVLDB* 13, 2 (2019), 184–196.

[26] Lukasz Golab, Howard Karloff, Flip Korn, Divesh Srivastava, and Bei Yu. 2008. On generating near-optimal tableaux for conditional functional dependencies. *VLDB* (2008).

[27] Han He. 2020. *HanLP: Han Language Processing*. https://github.com/hankcs/HanLP

[28] Alireza Heidari, Joshua McGrath, Ihab F. Ilyas, and Theodoros Rekatsinas. 2019. HoloDetect: Few-Shot Learning for Error Detection. In *SIGMOD*.

[29] Xuegang Hu and Haitao Yu. 2006. The Research of Sampling for Mining Frequent Itemsets. In *Rough Sets and Knowledge Technology (RSKT)*.

[30] Ykä Huhtala, Juha Kärkkäinen, Pasi Porkka, and Hannu Toivonen. 1999. TANE: An Efficient Algorithm for Discovering Functional and Approximate Dependencies. *Comput. J.* (1999).

[31] Wontae Hwang and Dongseung Kim. 2006. Improved Association Rule Mining by Modified Trimming. In *Computer and Information Technology (CIT)*.

[32] Caiyan Jia and Ruqian Lu. 2005. Sampling Ensembles for Frequent Patterns. In *Fuzzy Systems and Knowledge Discovery (FSKD)*.

[33] Hanna Köpcke, Andreas Thor, and Erhard Rahm. 2010. Evaluation of entity resolution approaches on real-world match problems. *PVLDB* 3, 1-2 (2010), 484–493.

[34] Ioannis Koumarelas, Thorsten Papenbrock, and Felix Naumann. 2020. MDedup: Duplicate detection with matching dependencies. *PVLDB* 13, 5 (2020), 712–725.

[35] Sebastian Kruse and Felix Naumann. 2018. Efficient discovery of approximate dependencies. *VLDB* (2018).

[36] Clyde P Kruskal, Larry Rudolph, and Marc Snir. 1990. A complexity theory of efficient parallel algorithms. *TCS* (1990).

[37] Bing Li, Yukai Miao, Yaoshu Wang, Yifang Sun, and Wei Wang. 2021. Improving the Efficiency and Effectiveness for BERT-based Entity Resolution. In *AAAI*. AAAI Press, 13226–13233.

[38] Feifei Li, Bin Wu, Ke Yi, and Zhuoyue Zhao. 2019. Wander Join and XDB: Online Aggregation via Random Walks. *TODS* 44, 1 (2019), 2:1–2:41.

[39] Weibang Li, Zhanhuai Li, Qun Chen, Tao Jiang, and Hailong Liu. 2015. Discovering functional dependencies in vertically distributed big data. In *WISE*. 199–207.

[40] Weibang Li, Zhanhuai Li, Qun Chen, Tao Jiang, and Zhilei Yin. 2016. Discovering approximate functional dependencies from distributed big data. In *APWeb*. 289–301.

[41] Yanrong Li and Raj P. Gopalan. 2004. Effective Sampling for Mining Association Rules. In *Advances in Artificial Intelligence*.

[42] Yanrong Li and Raj P. Gopalan. 2005. Stratified Sampling for Association Rules Mining. In *Artificial Intelligence Applications and Innovations (IFIP)*.

[43] Yuliang Li, Jinfeng Li, Yoshihiko Suhara, AnHai Doan, and Wang-Chiew Tan. 2020. Deep Entity Matching with Pre-Trained Language Models. *arXiv preprint arXiv:2004.00584* (2020).

[44] Ester Livshits, Alireza Heidari, Ihab F. Ilyas, and Benny Kimelfeld. 2020. Approximate Denial Constraints. *PVLDB* 13, 10 (2020), 1682–1695.

[45] Stéphane Lopes, Jean-Marc Petit, and Lotfi Lakhal. 2000. Efficient discovery of functional dependencies and Armstrong relations. In *EDBT*. Springer, 350–364.

[46] Basel A. Mahafzah, Amer F. Al-Badarneh, and Mohammed Z. Zakaria. 2009. A new sampling technique for association rule mining. *J. Inf. Sci.* 35, 3 (2009), 358–376.

[47] Heikki Mannila, Hannu Toivonen, and A. Inkeri Verkamo. 1994. Efficient Algorithms for Discovering Association Rules. In *Knowledge Discovery in Databases*.

[48] Volodymyr Mnih, Koray Kavukcuoglu, David Silver, Andrei A. Rusu, Joel Veness, Marc G. Bellemare, Alex Graves, Martin A. Riedmiller, Andreas Fidjeland, Georg Ostrovski, Stig Petersen, Charles Beattie, Amir Sadik, Ioannis Antonoglou, Helen King, Dharshan Kumaran, Daan Wierstra, Shane Legg, and Demis Hassabis. 2015. Human-level control through deep reinforcement learning. *Nat.* (2015).

[49] MEJ Newman. 2005. Power laws, Pareto distributions and Zipf's law. *Contemporary Physics* 46, 5 (sep 2005), 323–351.

[50] Noel Novelli and Rosine Cicchetti. 2001. Fun: An efficient algorithm for mining functional and embedded dependencies. In *ICDT*. Springer, 189–203.

[51] Thorsten Papenbrock and Felix Naumann. 2016. A Hybrid Approach to Functional Dependency Discovery. In *SIGMOD*.

[52] Srinivasan Parthasarathy. 2002. Efficient Progressive Sampling for Association Rules. In *International Conference on Data Mining (ICDM)*. 354–361.

[53] Eduardo H. M. Pena, Eduardo Cunha de Almeida, and Felix Naumann. 2019. Discovery of Approximate (and Exact) Denial Constraints. *PVLDB* 13, 3 (2019), 266–278.

[54] Abdulhakim Qahtan, Nan Tang, Mourad Ouzzani, Yang Cao, and Michael Stonebraker. 2020. Pattern functional dependencies for data cleaning. *PVLDB* 13, 5 (2020), 684–697.

[55] Nils Reimers and Iryna Gurevych. 2019. Sentence-BERT: Sentence Embeddings using Siamese BERT-Networks. *CoRR* abs/1908.10084 (2019). arXiv:1908.10084

[56] Theodoros Rekatsinas, Xu Chu, Ihab F. Ilyas, and Christopher Ré. 2017. HoloClean: Holistic Data Repairs with Probabilistic Inference. *PVLDB* (2017).

[57] Matteo Riondato and Eli Upfal. 2015. Mining Frequent Itemsets through Progressive Sampling with Rademacher Averages. In *SIGKDD*.

[58] Hemant Saxena, Lukasz Golab, and Ihab F Ilyas. 2019. Distributed discovery of functional dependencies. In *ICDE*. IEEE, 1590–1593.

[59] Hemant Saxena, Lukasz Golab, and Ihab F Ilyas. 2019. Distributed implementations of dependency discovery algorithms. *PVLDB* 12, 11 (2019), 1624–1636.

[60] Philipp Schirmer, Thorsten Papenbrock, Ioannis Koumarelas, and Felix Naumann. 2020. Efficient Discovery of Matching Dependencies. *TODS* 45, 3 (2020), 1–33.

[61] Philipp Schirmer, Thorsten Papenbrock, Sebastian Kruse, Felix Naumann, Dennis Hempfing, Torben Mayer, and Daniel Neuschäfer-Rube. 2019. DynFD: Functional Dependency Discovery in Dynamic Datasets. In *EDBT*.

[62] Michael Sejr Schlichtkrull, Thomas N. Kipf, Peter Bloem, Rianne van den Berg, Ivan Titov, and Max Welling. 2018. Modeling Relational Data with Graph Convolutional Networks. In *ESWC*.

[63] Rohit Singh, Venkata Vamsikrishna Meduri, Ahmed K. Elmagarmid, Samuel Madden, Paolo Papotti, Jorge-Arnulfo Quiané-Ruiz, Armando Solar-Lezama, and Nan Tang. 2017. Synthesizing Entity Matching Rules by Examples. *PVLDB* 11, 2 (2017), 189–202.

[64] Shaoxu Song and Lei Chen. 2009. Discovering matching dependencies. In *CIKM*.

[65] Jie Tang, Jing Zhang, Limin Yao, Juanzi Li, Li Zhang, and Zhong Su. 2008. ArnetMiner: Extraction and Mining of Academic Social Networks. In *KDD'08*. 990–998.

[66] Benjamin Taskar, Ming Fai Wong, Pieter Abbeel, and Daphne Koller. 2003. Link Prediction in Relational Data. In *NIPS*. 659–666.

[67] Hannu Toivonen. 1996. Sampling Large Databases for Association Rules. In *VLDB*. Morgan Kaufmann, 134–145.

[68] Leslie G. Valiant. 1990. A Bridging Model for Parallel Computation. *Commun.*

ACM 33, 8 (1990), 103–111.

[69] Catharine M. Wyss, Chris Giannella, and Edward L. Robertson. 2001. FastFDs: A Heuristic-Driven, Depth-First Algorithm for Mining Functional Dependencies from Relation Instances - Extended Abstract. In *DaWak*.

[70] Ying Yan, Liang Jeff Chen, and Zheng Zhang. 2014. Error-bounded Sampling for Analytics on Big Sparse Data. *PVLDB* (2014).

[71] H Yao, H Hamilton, and C Butz. 2002. Fd_mine: Discovering functional dependencies in a database using equivalences. In *IEEE ICDM*. 1–15.

[72] Juntao Yu, Bernd Bohnet, and Massimo Poesio. 2020. Named Entity Recognition as Dependency Parsing. In *ACL*.

[73] Chengqi Zhang, Shichao Zhang, and Geoffrey I. Webb. 2003. Identifying Approximate Itemsets of Interest in Large Databases. *Appl. Intell.* 18, 1 (2003), 91–104.

[74] Yunjia Zhang, Zhihan Guo, and Theodoros Rekatsinas. 2020. A Statistical Perspective on Discovering Functional Dependencies in Noisy Data. In *SIGMOD*. 861–876.

[75] Yanchang Zhao, Chengqi Zhang, and Shichao Zhang. 2006. Efficient Frequent Itemsets Mining by Sampling. In *Advances in Intelligent (IT)*.